Moody's Approach To Rating Synthetic CDOs

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OVERVIEW

Synthetic CDOs have gained tremendous popularity since their initial appearance in 1997. In Europe, where the synthetic CDO market is more established, synthetic CDOs accounted for 85% (by number) of all CDOs rated by Moody’s in 2002. In the U.S., they have grown to account for nearly 30% of all CDOs rated by Moody’s in 2002, up significantly from approximately 13% in 2001. ¹ While these structures share some similarities with conventional cash flow CDOs, they have several dis-


* Jeffrey Tolk and Myles Bae contributed to this report while serving as analysts within Moody’s Derivatives team. Jeffrey is now at HSBC Bank and Myles is now at BNP Paribas.
tistinguishing features that warrant some modification to Moody's traditional CDO rating methodology. The purpose of this special comment is to describe those features and to explain Moody's approach to rating synthetic CDOs.

**WHAT IS A SYNTHETIC CDO?**

Within the realm of synthetic CDOs, there is much variation among structures. They differ by underlying motivation (balance sheet vs. arbitrage), collateral management (static vs. managed), liability structure (single tranche vs. whole capital structure) and liability distribution (funded vs. unfunded), as well as by reference asset type. The common thread among all of these structures is the synthetic nature of the credit risk exposure. Unlike cash flow CDOs that own the cash position in a bond, loan or other security, synthetic CDOs access this exposure through the use of credit derivatives such as credit default swaps.

In a typical credit default swap (see Figure 1), the "seller" of credit protection assumes the risk of default of a reference entity or reference credit from the "buyer" of credit protection in return for a periodic premium payment. If the reference entity/credit should default according to the terms of the credit default swap documentation (a "credit event"), the seller pays the buyer a protection payment.

If the credit default swap documentation specifies cash settlement, the reference credit is valued and the seller pays the buyer an amount equal to the original notional amount of the credit default swap minus the market value of the reference credit. If physical settlement is specified, the seller pays the buyer the full notional amount of the trade and is delivered an asset of the reference entity that meets certain requirements specified in the credit default swap documentation.

In synthetic CDOs, the CDO investor effectively acts as the seller of credit protection on a pool of reference entities/credits to a single counterparty or multiple counterparties (the buyers of credit protection).  

**Figure 1**  
Typical Credit Default Swap

Balance Sheet Synthetic CDOs

In a typical balance sheet synthetic CDO structure, there is a sponsoring financial institution (the buyer of credit protection) that uses a credit default swap to remove credit exposure from its balance sheet while retaining ownership of the assets. In this way, the financial institution is able to manage its credit risk more efficiently and to obtain economic and/or regulatory capital relief while maintaining its relationship with the reference entities.

Balance sheet CDOs are typically static or lightly managed vehicles where there is little to no ability for the financial institution to substitute new reference entities/credits once the transaction commences.

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2 This special report only addresses synthetic CDOs that reference corporate entities. It does not cover synthetic CDOs that reference structured finance assets ("synthetic resecuritizations"). Synthetic resecuritizations have their own unique features that require a separate discussion. Moody's expects to publish a special report in the near future that addresses these features and the analyses required to rate this specialized form of synthetic CDO.

3 If no reference credit has been previously identified, the protection buyer has the option of choosing any credit of the reference entity that meets the specific terms of the credit default swap agreement as the reference credit - usually a senior unsecured bond or loan.

4 See Moody's Special Report, "Synthetic CDO's: European Credit Risk Transfer 'A La Carte,'" July 2000 for an earlier discussion about synthetic CDOs.

5 "Lightly managed" vehicles generally allow a limited number of substitutions or allow for the deletion of reference entities/credits for credit risk purposes.
The credit risk from the reference pool is tranched and sold to investors with the financial institution typically retaining the first loss position (the "equity"). There is also often a "super-senior" piece (the last loss position) that is either retained by the financial institution or transferred to other counterparties by way of credit default swaps. As losses occur in excess of the retained first loss position, the financial institution is reimbursed for the losses incurred on its balance sheet.

**Arbitrage Synthetic CDOs**

Arbitrage synthetic CDOs, which developed a little later than balance sheet synthetic CDOs, take advantage of the difference between the spread received from selling protection on individual reference entities/credits and the spread paid to investors to buy protection on a pooled basis. Like their arbitrage cash flow counterparts, excess spread plays an important part in the mechanisms of many of these transactions. This excess spread may be trapped to offset previous losses, used to enter into additional trades or hedging agreements, or treated as returns to equity investors.

Arbitrage synthetic CDOs, which tend to be more structurally complicated than balance sheet CDOs, may allow for active management of the reference pool. Generally, an investment advisor is engaged to make substitution and/or hedging decisions on behalf of the CDO within the strict guidelines of the deal's governing documents (e.g., the indenture, the swap confirmation, etc.).

In some of these transactions, there may be multiple swap agreements with multiple counterparties or buyers of credit protection. Also, there may be provisions that allow the CDO to buy protection for hedging or arbitrage purposes. The capital structures for arbitrage synthetic CDOs look very similar to balance sheet synthetic CDOs in that there is a tranching of the credit risk exposure. In arbitrage synthetic CDOs, however, the first loss position is sometimes sold in its entirety to third party investors. In some structures, the equity is not placed at all and is created internally through the transactions' mechanisms for allocating excess spread to absorb losses up to a certain threshold.

More recently, a new type of arbitrage synthetic CDO has become very popular. The structure allows the CDO investor to choose the reference pool and the loss position in the capital structure. These single-tranche, reverse-inquiry, "bespoke" transactions are generally static in nature, and therefore, forego the necessity and cost of engaging an investment advisor (although more and more, these structures engage an investment advisor to allow for defensive trading). The structuring institution bears the responsibility for hedging the loss positions above and below the investor's tranche and generally employs a portfolio hedging strategy to do so. This structure allows investors to choose reference entities with which they are most comfortable while customizing the risk subordination to their specifications.

**Funded vs. Unfunded Synthetic CDOs**

All of the synthetic CDO types discussed above are structured in such a way that the investors take either a funded or unfunded position in the transactions. In other words, depending on the CDO, there may be an option for the investor to purchase CDO notes at the beginning of the transaction ("funded") or to make payments as losses occur in the reference pool ("unfunded").

In a typical unfunded synthetic CDO (see Figure 2), the protection buyer enters into a credit default swap with the issuer, a special purpose vehicle ("SPV" or the "issuer"), which in turn, enters into credit default swaps with the individual investors, the ultimate protection sellers. In many cases, especially among the single-tranche arbitrage synthetic CDOs described above, the SPV may be eliminated and the credit default swap agreements drawn directly between the counterparty and the investors.

No cash is exchanged at the beginning of the transaction. A periodic swap premium is paid by the counterparty and passed through to the investors on a priority basis. In other words, the senior-most investors receive their promised spread first, then the next most senior investors, and so on. To the extent that losses reduce the notional amount of the reference pool and if the counterparty does not pay a fixed spread on the credit default swap, the premium may be insufficient to pay the equity and junior-most tranches.

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6 While these single tranche transactions have existed in Europe for approximately 2 years, they have only recently become popular in the United States where full capital structure synthetic CDOs were more prevalent until the latter part of 2002.

7 They may also forego the expertise that a good and experienced investment advisor may bring to a transaction.
As losses occur, protection payments are made to the counterparty by the investors in a reverse priority order. Depending on the structure, excess spread may be used to reinstate a portion of the first loss position or a junior tranche that has incurred losses. While some structures stop making spread payments to investors on the notional amounts that have been written down, others may continue to make spread payments on the initial notional amounts of the CDO tranches regardless of the principal loss. This is especially true of structures that make all protection payments at maturity.

The typical structure for funded synthetic CDOs is slightly more complicated (see Figure 3). In these CDOs, the cash proceeds from the sale of notes by the issuer are usually invested in highly liquid, high-quality eligible investments (the "collateral"). The coupon paid to the note investors is comprised of the swap premium received from the counterparty and the interest proceeds received from the collateral. If losses occur in the reference pool, collateral is liquidated (or put at par to the arranger/swap counterparty - this is generally the case for less liquid collateral or to minimize O/C requirements on the collateral) to pay the counterparty the protection payments. Depending on the structure, excess spread may be used to purchase additional collateral to compensate for losses.

At the maturity of the transaction, the remaining collateral is liquidated (or put at par to the initial arranger/swap counterparty - this is usually the case with collateral that have longer maturities than the swap) and the resulting funds are used to repay principal to the note holders on a priority basis. If, at any time, losses exceed the amount remaining in the collateral account, the counterparty may have recourse to the super-senior counterparty(s) (usually an unfunded position) for the remaining losses generated by the reference pool.
In many synthetic CDOs (both funded and unfunded), the counterparties pay a pre-set swap premium to the CDO investors regardless of the actual spreads implied by the reference pool. This premium is calculated to ensure coverage of all interest payments on the CDO. In other words, there is no excess spread in the CDO, nor is there a weighted average spread covenant. In these transactions, the principal risk still comes from the reference pool; although, the spread risk comes only from the counterparty.

**MOODY’S RATINGS FRAMEWORK**

Moody’s rating on each rated note represents our opinion of the expected loss on the note, which is the difference between the present value of the expected payments on the note and the present value of the promised payments under the note, expressed as a percentage of the present value of the promise.

For unfunded CDOs, this essentially translates to the expected loss posed to the investor relative to the present value of the promised swap premium payments and the requirement to make any credit protection payments to the counterparty.

To evaluate the expected loss, Moody’s applies an approach that incorporates both quantitative and qualitative analyses. All potentially material risks to a CDO must be either quantitatively modeled or structurally mitigated. Moody’s expected loss models capture the quantifiable risks while a legal review of the transaction seeks to ensure that non-quantifiable risks are mitigated through documentation provisions. Moody’s strives to provide comprehensive coverage of a CDO’s risks through this analytical framework.

**THE QUANTITATIVE ANALYSIS**

The primary source of risk in a synthetic CDO comes from the reference pool. CDO investors sustain losses as credit events erode the notional amount of the reference pool requiring protection payments to the counterparty.

Regardless of the synthetic CDO type, the same basic quantitative analysis may be used to assess the risks stemming from the reference pool.

Structures where the counterparty pays a set swap premium may require an even simpler analysis focusing only on principal losses. In other words, the premium payments may be excluded from the scope of the quantitative analysis because the promised premium is large enough to ensure coverage of the interest payments on the CDO. To mitigate the risk of non-payment of the premium, the governing documentation of the transaction gen-
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Generally contains a provision wherein the counterparty collateralizes the swap premium or pays the premium up front (rather than in arrears) when certain counterparty rating requirements are not met. If such a provision is not included in the documentation, Moody's may model the risk of non-payment of premium in the event of a counterparty default.

Depending on the specific CDO structure, the counterparty and/or collateral may contribute additional risks to the synthetic CDO transaction, but these risks may be mitigated or eliminated through documentation provisions or modeled through modifications to the quantitative approach described below. This special report will not cover in detail such secondary risks to the CDO investor.8

The Binomial Expansion Method - A Review: Modeling Default Likelihood

Moody's traditional CDO modeling methodology captures the default risk for a pool of assets through the application of the Binomial Expansion Method.9 We feel that compared to other more complex modeling techniques such as simulation-based analyses, the Binomial Expansion Method applied to a diverse pool of relatively homogeneous assets produces the most transparent, yet accurate assessment of expected loss. By adding recovery assumptions and cash flow guidelines from the CDO's governing priority of payments, we can assess the expected loss for any tranche of a CDO.

To use the Binomial Expansion Method, a model portfolio is created which contains N number of idealized securities ("diversity bonds"). Each diversity bond is assumed to have identical characteristics in terms of par/notional amount, rating, average life, spread and recovery, and is uncorrelated with every other diversity bond in the pool. In other words, each diversity bond has the same probability of default and the same recovery in the event of default. The default likelihood of any single diversity bond is independent of the default of any other diversity bond in the model portfolio.

The characteristics assigned to the diversity bonds reflect the average characteristics of the actual portfolio (e.g., average rating factor, recovery, spread, etc.). The number of diversity bonds in the portfolio is equivalent to Moody's diversity score, which is calculated by taking into account the issuer and industry concentrations in the actual portfolio and by applying default correlations among the various assets.

For a static CDO, the actual characteristics of the portfolio are used to create the model portfolio. For a managed CDO, the portfolio constraints specified in the governing documents are used to create the model portfolio, since trading may quickly transform the initial portfolio profile.

The Binomial Expansion Method examines the losses stemming from the default of each additional diversity bond in the model portfolio going from zero diversity bond defaults to N diversity bond defaults and assigns a probability to each default scenario. Calculating this probability-weighted loss for each CDO tranche generates the expected loss.

The Multiple Binomial: Modeling Default Likelihood in Certain Circumstances

For cash flow CDOs, the Double Binomial Method10 has been used in cases where the underlying portfolio assets exhibit heterogeneous characteristics - such as having a clear delineation between low rated and highly rated assets. For such portfolios, the average characteristics of the asset pool used to describe the model portfolio are not representative of the actual loss distribution of the permissible portfolio. In these cases, the model portfolio must be broken out into two sub-model portfolios, each of which has its own average characteristics that are representative of the specific sub-portfolio, and a binomial analysis is run on both. Generally, the double binomial method is used when there is a wide or barbelled distribution in ratings or any other characteristic among the assets in a portfolio. In these cases, the average does not capture the true risk of the portfolio's distribution.

For many synthetic CDOs, Moody's takes the Double Binomial Method one step further by dividing a pool of reference entities/credits into the most appropriate number of sub-pools and by modeling the default behavior of each pool with a separate binomial analysis. Whereas in a double binomial, there are only two pools, the multiple binomial analyses may use any number of pools. There are several aspects of a synthetic CDO that, when

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8 See Moody's Special Report "Moody's Approach to Assessing Secondary Risks in Synthetic CDOs," March 2003 for a full discussion of these secondary risks stemming from the counterparties and collateral.
viewed in combination, may warrant the use of the Multiple Binomial Method, to help quantify the inherent risks. These include portfolio characteristics, capital structure, and structural features as explained below. These characteristics are also the primary determinants of the number of pools to use in the multiple binomial analysis.

**Portfolio Characteristics: Looking at the Range of Credit Risks in the Reference Pool**

Most synthetic CDOs have reference entities/credits whose ratings can vary greatly (typically Aaa down to Baa3 or even Ba3). For a 5-year synthetic CDO, Moody’s idealized default probability can vary from as little as 0.003% for a Aaa credit to 3.05% for a Baa3 credit and 11.86% for a Ba3 credit.

Consider fifty independent five-year diversity bonds: forty $1.25 million bonds rated A3 (default probability 0.73%), five $5.0 million bonds rated Baa3 (default probability 3.05%) and five $1.0 million bonds rated Ba1 (default probability 5.28%). Table 1 lists the cumulative default probability distribution for both a single binomial and a triple binomial using a weighted average default probability of 1.74% (approximately Baa2/Baa3) and a total diversity score of 50 (see Appendix A for the Moody’s Idealized Default Probability table).11

<table>
<thead>
<tr>
<th># of Defaults</th>
<th>% of Defaults</th>
<th>Cumulative Probability Single Binomial</th>
<th>Cumulative Probability Triple Binomial</th>
<th>Tail Probability Single Binomial</th>
<th>Tail Probability Triple Binomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00%</td>
<td>41.59%</td>
<td>48.72%</td>
<td>58.41%</td>
<td>51.28%</td>
</tr>
<tr>
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<td>2.00%</td>
<td>78.40%</td>
<td>76.62%</td>
<td>21.60%</td>
<td>23.38%</td>
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<tr>
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<td>94.36%</td>
<td>84.27%</td>
<td>5.64%</td>
<td>15.73%</td>
</tr>
<tr>
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<td>6.00%</td>
<td>98.88%</td>
<td>85.62%</td>
<td>1.12%</td>
<td>14.38%</td>
</tr>
<tr>
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<td>97.70%</td>
<td>0.18%</td>
<td>2.30%</td>
</tr>
<tr>
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<td>1.09%</td>
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<td>0.00%</td>
<td>0.89%</td>
</tr>
<tr>
<td>7</td>
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<td>100.00%</td>
<td>99.74%</td>
<td>0.00%</td>
<td>0.26%</td>
</tr>
<tr>
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<td>16.00%</td>
<td>100.00%</td>
<td>99.96%</td>
<td>0.00%</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

* Tail probability = 1 - Cumulative Probability

In the case of the single binomial, only one pool was created with a diversity score of 50 and a default probability equivalent to the pool average.

For the triple binomial, three pools were created with the diversities and default probabilities associated with each respective rating level.

Table 1 also shows the tail probability (T(x)) in each default scenario where T(x) = 1 minus the cumulative default probability or the probability of more than x defaults occurring. The expected loss calculation can be dependent on the losses that occur at the tail of the loss distribution curve. By simplifying the analysis into the single binomial, a 12% default scenario (6 out of 50 diversity bonds default) is expected to capture the entire loss distribution curve. By dividing the pool into its components, however, the triple binomial more accurately shows that there may be material losses that need to be accounted for in default scenarios that exceed 16% (8 out of 50 diversity bonds default).12

Not all portfolio distributions require the use of the Multiple Binomial Method; however, when there is enough variance in the reference pool components, the single Binomial Expansion Method may not be sufficient to correctly capture the loss distribution of the reference pool. Additionally, a miscalculation of the tail probability can be exacerbated by the capital structures used by synthetic CDOs, as explained below.

**Capital Structure: Accounting for Tail Default Probabilities**

Most synthetic CDOs are highly leveraged and are thus sensitive to fewer defaults than cash flow CDOs. Because the reference pools tend to be highly rated on average and the transactions tend to have relatively...
short maturities (usually five years), only a small amount of subordination is necessary to support high ratings. This thin subordination combined with the relatively small sizes of the rated tranches generally requires more precision in the calculation of the tail probability of the loss distribution.

It is important to note that by expanding the tail probability, more of the rated notes may be affected materially. In many default scenarios under both the single and multiple binomial, the equity and more junior classes will be completely exhausted. However, by expanding the tail probability, losses may reach further up the capital structure to the mezzanine and senior notes.

Take our single vs. triple binomial example above. Let's assume that the capital structure associated with that reference pool looks like the one in Figure 4. Let's also assume that the reference entities/credits all have a recovery rate of 30%.

In Figure 4, the Class B investor has attachment points between 7% and 10%. In other words, as losses eat their way through the capital structure from the bottom up, the Class B investor absorbs any losses above 7% of the reference pool up to 10% of the reference pool. Using the single Binomial Expansion Method, the expected loss for the Class B tranche is 0.0238% (compared to a Aa2 5-year expected loss of 0.0374%). Using the Multiple Binomial Method, the expected loss for the Class B tranche is 0.4432% (compared to a Baa1 5-year expected loss of 0.6050%).13 In this simplified and extreme example, there is as much as a five-notch differential in the rating using the different methodologies (See Appendix B for Moody's Idealized Expected Loss table). If Moody's were rating this structure, the Class B tranche would be assigned a rating of Baa1.

Due to the highly leveraged nature of most synthetic CDOs, only a small group of the lowest rated credits can have a large impact on the loss distribution, and therefore, the ratings. This disparity in potential outcomes on the CDO liabilities may be as small as one notch. However, if the dispersion of the ratings in the reference portfolio around the average is large enough (and especially if there is a non-investment grade basket as in our example), this disparity may be upwards of four or five notches.

**Structural Features, Or Lack Thereof: Contributors to the Importance of the Multiple Binomial**

While some of the newer synthetic CDOs incorporate features such as an overcollateralization test ("O/C test") or interest coverage test ("I/C Test") - both of which use excess spread to reduce leverage or build up reserve accounts - many synthetic CDOs still do not have these features.

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13 In this simplified example, expected loss is calculated only in respect to principal. Moody's generally calculates expected loss based on the present value of all promised cash flows. The exact relationship between the losses in the reference pool and the losses to the liabilities will depend on the details of the cash flow waterfall of the CDO.
As discussed earlier, many synthetic CDOs do not even have the ability to generate any excess spread that may be used to offset losses in the reference pool. A CDO that does not incorporate such structural features to mitigate some of the concerns illustrated above will feel the immediate impact of losses to the reference pool. In fact, in many CDOs, the principal amount of the tranches may be immediately and permanently written-down (in reverse priority order) as losses occur.

In these cases, it is even more important to capture the correct loss distribution when analyzing the expected loss of a CDO tranche. It should be noted that while many market participants prefer the use of excess spread to mitigate the effects of losses in the reference pool, all else being equal, it makes little difference from the ratings point of view. The inclusion or exclusion of excess spread in a synthetic CDO would be modeled either way with the result being different capital structures.

The following is a mathematical expression of the multiple binomial-based expected loss calculation that would have been used by Moody's in rating the example given in the section above:

$$E(L) = \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} p_{ijk} L_{ijk}$$

where

- \( i \) = Number of diversity bond defaults in \( A3 \) pool
- \( j \) = Number of diversity bond defaults in \( Baa3 \) pool
- \( k \) = Number of diversity bond defaults in \( Ba1 \) pool
- \( l \) = Diversity score of \( A3 \) pool
- \( m \) = Diversity score of \( Baa3 \) pool
- \( n \) = Diversity score of \( Ba1 \) pool
- \( p_{ijk} \) = Probability of having \( i, j, \) and \( k \) number of defaults in \( A3, Baa3 \) and \( Ba1 \) pool, respectively
- \( L_{ijk} \) = Loss to the CDO liability when there is \( i, j, \) and \( k \) number of defaults in \( A3, Baa3 \) and \( Ba1 \) pool, respectively

Again, the number of binomial pools to use for modeling a synthetic CDO depends on a combination of the portfolio characteristics (or portfolio guidelines for a managed CDO), the capital structure of the CDO and other structural features incorporated into the transaction.

At first, this may just be a matter of trial and error. Some structurers and rating analysts automatically divide the pool into each rating category represented in the CDO. At the same time, we must be careful not to create binomial pools that are too small (e.g., diversity scores of 1 or 2), which may have a negative effect on the accuracy of the analysis. The general rule is: if creating finer allocations (within certain size constraints) affects the expected loss of a tranche, the additional binomial pools are necessary. In some cases, a single binomial may be sufficient at the outset of the CDO, but as migration creates a more widely dispersed portfolio, a multiple binomial may become necessary for monitoring purposes.

**Diversity Allocation: Scaling the Diversity Score**

One of the biggest challenges in employing the Multiple Binomial Method is ensuring that the overall diversity score is correctly allocated to each rating basket used in modeling. Usually, the sum of the diversity scores calculated for each individual pool will exceed the diversity score calculated for the overall pool as a whole. This is due to the fact that there is generally correlation among the individual pools. One of the underlying assumptions in performing a multiple binomial analysis is that each pool defaults independently of the others (i.e., all of the pools are uncorrelated). Therefore, the unadjusted diversity scores for the individual pools are usually scaled down such that the sum of the "normalized" or "adjusted" diversity scores is equivalent to the overall pool diversity score.\(^{14}\)

Figure 5 illustrates a simple, but very useful allocation scheme, which we will refer to as "linear diversity scaling." Please note that Figure 5 is only an example. Moody's reviews all characteristics of each reference pool carefully and performs extensive analyses to test the accuracy and sensitivity of our modeling assumptions.

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\(^{14}\) In most cases where ratings are the determinant of the binomial pool break-down, a normalization is required, but there may be some cases where the individual binomial pools are completely uncorrelated. In these cases, a scaling-down of the diversity scores is unnecessary. In some rare cases, especially where there is complete heterogeneity in par and industry as well as rating, the sum of the individual diversity scores may be less than the total diversity score (i.e., \( D1 + D2 < D \)). In such cases, we would cap the normalization factor at 1.
One important consideration in this analysis is the preservation of the standard deviation of the default distribution implied by the overall pool diversity. The linear or pro rata diversity scaling method described in Figure 5 is an approximation of the appropriate allocation. If the overall pool variance is reduced when performing this simple linear diversity scaling, an alternative scaling method should be used, which takes into account both the mean and the standard deviation of the default distribution. A couple alternative methods are described in Appendix C.

**The Par Per Diversity Test**

The example in Figure 5 is useful in the context of a static transaction. However, in a managed synthetic CDO, where the reference pool profile may change over time, there should be controls embedded in the transaction to ensure that the modeled par and diversity allocations are maintained. If, for example, the Baa3 basket has a 10% maximum limitation, which was modeled at a diversity of 15, we would expect that if the basket is fully used, the scaled diversity score of the basket is at least 15.

However, it does not make sense to apply an absolute minimum diversity constraint on the basket of 15. What if the transaction only had 2% in the Baa3 basket? It would be impossible, not to mention unnecessary, to maintain this minimum diversity score.

Using the percentage limitations per basket as well as the guidelines for issuer and industry concentrations specified in the governing documentation, it is possible to create a “worst case” portfolio profile. This is the portfolio that will yield the worst expected loss. If the scaled diversities calculated for each pool under this “worst case” profile yield the expected loss necessary to obtain the target ratings, then no additional constraints are necessary. However, in many synthetic CDOs, this is not the case. One or more of the lower-rated baskets generally requires a higher diversity score than is implied by the documentation guidelines. If this is the case, either the portfolio guidelines will be modified to yield the appropriate diversity score or a “Par Per Diversity” test is generally included in the CDO in order to maintain the relationship between par and diversity necessary to maintain the ratings.

The Par Per Diversity test is usually applied to all but the senior-most binomial pool. The test is a comparison of the actual ratio of the par in a specific basket to the scaled down diversity score of the basket versus the ratio used in modeling the transaction. The tests may be applied to baskets individually (e.g., one ratio for the Baa3 basket, another for the Baa2 basket, etc.) or cumulatively (e.g., one ratio for the Baa3 basket, another for the Baa2 and below basket, etc.). While overall industry and issuer concentration limits in a CDO may allow the transaction to have a lower diversity score in certain baskets, the Par Per Diversity test ensures that the diversity will not fall below the modeled levels.
Default Timing
In modeling a cash flow CDO, we vary the timing with which cumulative defaults occur for each scenario. Typically, the defaults would be spread over a six-year time horizon starting in year one with a default spike of 50% in one of the six years and the remaining 50% being distributed evenly among the remaining five years (i.e., 10% in each of the years). We would run six different expected loss analyses as we move the default spike through the six years.

The same concept applies for synthetic CDOs with a few minor differences. Most synthetic CDOs have a maturity of five years. To make sure that all of the reference pool is being allowed to default, we shorten the default horizon from six to five years. Thus, we only run five different default-timing analyses, wherein the remaining cumulative defaults (after the initial default spike is applied) are divided among four years rather than five.\(^{15}\)

One additional default-timing scenario we may run involves the marginal default probabilities for each multiple binomial category. Since we break the reference pool into specific rating categories, we have the ability, using Moody’s idealized default probability table (see Appendix A), to calculate not only each pool’s cumulative default probability over the five-year time horizon, but the marginal default probabilities during each of those years. Therefore, we can calculate and use in our models default timings based on the idealized marginal default probabilities for each Multiple Binomial category.\(^{16}\)

Beyond the Multiple Binomial
All binomial methods (Binomial Expansion Method, Double Binomial Method, and Multiple Binomial Method) utilize diversity bonds to capture potential defaults in a pool of assets; however, there are cases where using any one of the binomial methods is not sufficient to capture the risks inherent in the pool. This often applies to pools that have a very low overall diversity score or in which the transaction structure warrants detailed pool analysis - perhaps even at the level of each underlying asset in the pool. This type of analysis may be more difficult if the transaction in question has a trading pool unless the investment advisor can replace each asset that is removed with another that is virtually identical in its characteristics (i.e., rating, maturity, industry, etc.).

For a static pool, however, Moody’s can model defaults in the pool at each underlying asset level to help capture the risks inherent in the transaction. For example, to rate a synthetic CDO whose reference pool consists of 10 securities, Moody’s may construct a Monte Carlo simulation model that generates defaults on the reference assets using the characteristics of each asset and the asset correlations among them.

As both protection buyers and sellers become more sophisticated and the credit derivatives market becomes more efficient, Moody’s expects to see transactions brought to market that are tailored to meet the specific needs of protection buyers and/or sellers. Such new transactions may warrant more frequent use of the Monte Carlo simulation approach.

THE QUALITATIVE ANALYSIS
To the extent that risks inherent in a synthetic CDO are not or cannot be modeled quantitatively, they would be addressed through the legal documentation. To this end, Moody’s conducts a comprehensive review of the governing documents and legal structure of each synthetic CDO we rate to determine that there are no hidden risks or perverse incentives that have been incorporated into the transaction. For managed synthetic CDOs, Moody’s also conducts an operations review to assess the investment advisor’s capacity and ability to manage the CDO.\(^{17}\)

Many of the important aspects of the qualitative analysis unique to synthetic CDOs can be grouped into three main categories:

1. Trading guidelines for managed synthetic CDOs
2. Credit event definitions and their effects on the modeled default probabilities
3. Structural features such as valuation procedures and settlement mechanisms that affect recovery rate assumptions.

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15 As the reference pool is usually comprised of investment grade reference entities, the default spike may be somewhat lower than the 50% typically used in high-yield CDOs.
16 In structures where the swap premium is pre-set and there is no excess spread, the expected loss is almost independent of the timing of losses. Therefore, the various default timings will have only a limited impact because of present value effects.
Depending on the specific terms of the documentation, the quantitative modeling may have to be adjusted to take into account certain additional risks.

**Trading Guidelines for Managed Synthetic CDOs**

In managed synthetic CDOs, sponsoring financial institutions and investment advisors have the ability to make trades after the effective date of the transaction. The level of flexibility depends on details of the transaction. In the more restrictive managed CDOs, there is only the ability to remove or substitute a few reference entities/credits or to remove reference entities/credits that have migrated in credit quality such that they can be categorized as "credit risk" trades.

In the more flexible CDOs, the investment advisor has the ability to enter into hedging contracts to mitigate the credit risk of an earlier trade or to enter into credit default swaps as the buyer of protection for arbitrage purposes. In all of these cases, it is important to check that the proper quality tests and eligibility criteria are incorporated into the documentation and are modeled appropriately.

**Isolating Trading Losses**

In reviewing the trading guidelines for a synthetic CDO, it is critical to ensure that the only losses to the investors' principal can come from credit events or from credit risk trades, whether through hedging or from unwinding a position. Discretionary trades and credit improved trades should typically not result in losses to the collateral account in a funded CDO or in protection payments being made in an unfunded CDO.

As credit risk trades can be considered to be much like "early credit events" with higher recoveries than may be salvaged by waiting for the credit event to occur, they usually maintain the integrity of the quantitative models. Much like cash flow CDOs, the decision as to what is a credit risk trade (as well as what is a credit improved trade) is a subjective decision on the part of the investment advisor. However, if there is a one-notch downgrade on the senior notes and/or a two-notch downgrade on the mezzanine/junior notes, discretionary trading is terminated and the definition of credit risk and credit improved switches to objective measures (e.g., ratings-based or spread-based measurements). In many transactions, there is also a cumulative loss test that shuts off discretionary trading in the event cumulative losses exceed a certain level.

**Determining When Credit Risk Trades Add Risk or Strengthen the Deal**

In certain cases, Moody's may require that measures be in place to protect the CDO liabilities from excessive losses due to credit risk trading. If a trade that is terminated for credit risk reasons truly would have led to a credit event, the credit risk trade is beneficial. However, removing a trade at a loss when it would not have ended in a credit event (i.e., no loss) results in losses to investors that were not originally contemplated in the ratings analysis. If there are no mechanisms to trap excess spread or trading gains, each credit risk trade will result in an immediate loss to the CDO. Unlike a cash flow CDO that usually has an O/C test to limit such par losses, some synthetic CDOs may undergo a continuous "chipping away" of subordination without any countervailing mechanisms.

To keep such losses at a minimum, the definition of a credit risk trade may require pre-specified objective measures from the beginning of the transaction in addition to the investment advisor's judgment that with the passage of time the reference entity/obligation will experience a credit event. A loss- or ratings-based trigger to shut off credit risk trading, with the ability of the investors to turn the trading back on if they feel that it is warranted, is also recommended. Some synthetic CDOs also allow investors to approve specific credit risk trades if the investment advisor believes that the objective measures should be overridden. Moody's welcomes this investor involvement, but cautions that the trades could trigger larger losses than anticipated by the ratings on the CDO liabilities when providing this approval.

**Addressing Adverse Selection**

While Moody's certainly is in favor of the early removal of deteriorating credits from a portfolio, it is important to note what new trades are being added to the pool using the remaining notional amount (i.e., the original notional amount minus the loss amount) from the credit risk trade or the credit event settlement. Ideally, the remaining notional amount is also removed from the reference pool and used to lower the notional amount of the senior-most investor in an unfunded CDO or the collateral associated with the remaining notional amount is passed on intact to the investors in a funded CDO. However, if the transaction allows for replenishment of the
remaining notional amount by entering into a new trade, there should be - in addition to the normal portfolio guidelines and trading rules (e.g., "maintain or improve") - some features to address adverse selection.

One such feature is a restriction on the maximum spread allowed per reference credit. Much like deeply discounted securities in a cash flow CDO, proportionally wide spreads relative to a reference entity/credit's rating signal that a credit is perceived by the market to have additional risks, which may not be readily apparent from the rating alone.

An alternative would be to implement some mechanism that provides a disincentive for the investment advisor to invest in a trade with relatively wide spreads. Because the existence of excess spread is generally to the immediate advantage of the equity holder and the investment advisor (as an equity holder or through contingent management fees), some transactions have provided for reserve accounts that trap the excess spread until maturity. Other structures tie the contingent management fee directly to the default/loss performance of the reference pool. In both alternatives, excess spread cannot leak out of the deal until the final performance metrics are tallied or unless the deal is in good health. Finally, in some cases, the sponsor or investment advisor invests in a vertical slice of the capital structure in order to show that his interests are aligned with those of all investors in the CDO.

**Trapping Trading Gains**

In many managed synthetic CDOs, the investment advisor has the ability to turn trading gains into excess spread and can allow them to go directly to the equity or the investment advisor as a contingent management fee. There should be a mechanism first, for netting gains against losses and second, for trapping these trading gains in a reserve account up to a certain threshold to offset any subsequent losses. If the transaction is failing its quality tests (e.g., rating distribution, diversity, weighted average spread, etc.) or its OC/IC tests (if any) are failing, the gains should be trapped in the reserve account regardless of the threshold until the tests are back in compliance.

**Provisions for Short Positions**

Finally, when entering into "short" positions (or trades in which the CDO is the buyer of protection) for hedging purposes, the transaction should match all of the provisions of the "long" position to receive full credit for the hedge. The counterparty for both trades should be the same with the legal right of offset to avoid any counterparty risk implications.

Some transactions have a small basket for "naked short" positions, in which the CDO acts as the buyer of protection purely for arbitrage reasons (i.e., the CDO does not have a "long" position on the credit). To enter into these naked short positions, the CDO must be passing all of its quality tests and OC/IC tests and have enough cushion in its weighted average spread ("WAS") test to include the periodic spread to be paid to the counterparty in the "short" trade. Generally, there is a slightly higher WAS test for naked shorts.


Moody's special report, "Understanding the Risks in Credit Default Swaps," (March 2001) detailed Moody's primary concerns with the use of credit derivatives in CDOs. One of the basic assumptions underlying a synthetic CDO is that only the credit risk of a reference portfolio is passed through to investors. Therefore, when analyzing a synthetic CDO, Moody's seeks to ensure that this assumption is correct. Otherwise, investors may be subject to greater expected losses than those incorporated in Moody's ratings of the CDO's liabilities. If there are elements of the synthetic CDO that add to the default frequency and/or loss severity of the reference credits, these must be accounted for in the quantitative or qualitative analysis.

The March 2001 special report described numerous risk factors associated with credit derivatives, which could cause the assumption above to be incorrect. These factors included "soft" credit events (i.e., credit events that may not necessarily result in a true default of the reference entity), moral hazards and settlement issues (see the above-referenced report for a full description of the issues surrounding each factor). Since the publication of that report, there has been much progress in eliminating or mitigating the risks identified. As a result of numer-

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18 When replenishing the reference pool, "maintain or improve" criteria should be applied on the basis of a comparison of the reference pool as it is before the removal of the first trade and after the addition of the new reference obligation.

19 Such credits may be on watch for downgrade or on negative outlook - Moody's signals to the market that an issuer's rating has a relatively high likelihood of being downgraded.
To address the soft credit event and moral hazard issues identified in the March 2001 special report, ISDA incorporated the following three supplements to their 1999 Credit Derivatives Definitions (the "1999 Definitions"):

- The Restructuring Supplement dated May 11, 2001 (the "Restructuring Supplement"),
- The Supplement Relating to Convertible, Exchangeable or Accreting Obligations dated November 9, 2001, (the "Convertible Supplement") and
- The Supplement relating to Successor and Credit Events dated November 28, 2001 (the "Successor and Credit Event Supplement" and together with the Restructuring Supplement and the Convertible Supplement, the "Supplements").

The Supplements, when they were issued, addressed the majority of the soft credit event and moral hazard issues identified in the March 2001 special report. Most of the provisions in the supplements have been incorporated into the new 2003 Credit Derivatives Definitions (the "2003 Definitions," and together with the 1999 Definitions, the "Definitions")

**Bankruptcy is Clarified**

The Successor and Credit Event Supplement resolved two long-standing issues in the definition of bankruptcy. ISDA agreed to amend Section 4.2(b) of the 1999 Definitions to insert the phrase "in a judicial, regulatory or administrative proceeding or filing" after the phrase "fails or admits in writing." Additionally, ISDA deleted in its entirety the last clause of Section 4.2 of the 1999 Definitions: "or (i) takes any action in furtherance of, or indicating its consent to, approval of, or acquiescence in, any of the foregoing acts."

**Obligation Acceleration Eliminated Among Most Dealers**

Additionally, most dealers in the U.S. and Europe have agreed to eliminate obligation acceleration as a viable credit event from most synthetic CDOs. Those CDOs that still include obligation acceleration as a credit event have mitigated the additional risk of this soft credit event by including provisions such as restricting settlement to physical delivery. Another alternative used by dealers is the institution of a waiting period following the event to allow for potential waivers or cures of the acceleration. The obligation acceleration is not a valid credit event if a waiver or cure takes place during the waiting period or if another credit event (e.g., bankruptcy, failure to pay, restructuring) occurs.

**Restructuring is Clarified**

As for the restructuring credit event, there have been several updates to the definition since the March 2001 special report was released. As we believe that these changes deserve their own section, we will describe them in more detail herein.

**Cash Settlement Mechanisms Harmonized with Moody's Default Studies**

Most credit derivatives dealers have also agreed to implement settlement mechanisms for cash settled transactions that allow recovery rates to be calculated as per Moody's historical default studies. These mechanisms include a waiting period of at least 30 days for cash settlement and a more widespread procedure for obtaining bid quotations.

Despite this progress there are still some material risks in the drafting of certain credit event definitions and the valuation mechanisms used in synthetic CDOs. These risk factors are explored in some detail below.

**Credit Event Definitions and Default Probability Implications**

Moody's has always recognized bankruptcy, failure to pay and certain types of restructuring events to be defaults as captured by its historical default studies. However, as discussed in more detail in the March 2001 special report, Moody's found certain credit event provisions in ISDA's 1999 Definitions to be too broad in the sense that the 1999 Definitions designated as credit events items that would not be captured in Moody's database as defaults. If a synthetic CDO's documentation included these soft credit events, the default frequency of

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20 Moody's is now in the process of reviewing the 2003 Definitions. While this discussion may refer to some parts of the 2003 Definitions, it is not meant to be a complete review of the 2003 Definitions and should not be viewed as such. Moody's intends to publish our official comments and opinions on the 2003 Definitions in a separate Special Report.

the reference pool may be understated. In other words, the CDO could be subject to a higher default rate than the statistical description of the reference pool may indicate based on Moody's ratings.

While the Supplements, the 2003 Definitions and other work-arounds fashioned by market participants eliminated many of the soft credit events, some issues remain, especially regarding the restructuring credit event.

**Restructuring: Some Clarification, but Still Some Issues**

The Supplements and the 2003 Definitions have addressed parts of the restructuring definition, but potential risks continue to linger. Moody's believes that the changes have mitigated the moral hazard issue involved with bilateral and club transactions somewhat. The Restructuring Supplement mandated that restructuring could only be called a credit event if the obligation that is subject to the credit event is a "multiple holder obligation." In other words, more than three non-affiliated holders must hold the obligation and there must be a two-thirds voting requirement to enact a restructuring.

Additionally, the Successor and Credit Event Supplement required that a certificate signed by a managing director of the protection buyer be delivered with the credit event notice if the sole source of the publicly available information backing the credit event is the buyer.

In the case of the "change in currency" restructuring provision (Section 4.7(a)(v) of the Definitions), Moody's believes that the new language adopted by ISDA first through the Successor and Credit Events Supplement and now in the 2003 Definitions is sufficient to meet the definition of a default.

The definition, as drafted in the 1999 Definitions, encompassed "any change in the currency or composition of any payment of interest or principal." Moody's considered this to be too broad. The new definition has now been limited to include only changes "in the currency or composition of any payment of interest or principal to any currency which is not a Permitted Currency." Permitted Currency is defined as the legal tender of any G-7 or OECD country whose local currency long-term rating is rated at least Aaa by Moody's or AAA by either Standard & Poor's Corp. or Fitch. Moody's is comfortable that the updated definition restricts credit events under this restructuring clause only to those that result in "diminished financial obligations."

Section 4.7(a)(iv) of the Definitions, which addresses subordination, was also updated by the Restructuring Supplement. However, Moody's found that this supplement did not go far enough. In the supplement as well as in the 2003 Definitions, the clause has been clarified to restrict acceptable subordination events to those of a legal nature (not just structural as per the 1999 Definitions). However, to the extent that an economic loss is not suffered, the subordination may be considered akin to a downgrade rather than default. One possible solution is to add the language, "to the level of equity" at the end of the definition such that it reads, "a change in the ranking in priority of payment of any obligation, causing the subordination of such obligation to the level of equity."

More troubling to Moody's is Section 4.7(a)(iii) of the Definitions, which is "a postponement or other deferral of a date or dates for either (A) the payment or accrual of interest or (B) the payment of principal or premium." In other words, maturity lengthening of any sort may be considered a restructuring credit event even if the lengthening does not lead to a loss.

A couple of positive recent developments may lead to restructuring being eliminated completely from credit derivatives going forward. Several leading market participants have indicated that they will be moving to a two-credit event standard - bankruptcy and failure to pay. In addition, the Basel Committee was reported recently to have announced that it might relax its standards requiring banks to use restructuring as a credit event to receive capital relief.

**Consequences of Soft Credit Events**

To account for soft credit events, Moody's applies a stress to the default probabilities used to model the transaction. Again, this report does not include Moody's final response to the 2003 Definitions. For now, if the synthetic CDO documentation uses the 1999 Definitions and includes the restructuring credit event without any of the Supplements (i.e., unmodified restructuring), we will apply a 12.5% stress to the modeled default probability. If all of the Supplements are applied to the 1999 Definitions or if the 2003 Definitions are used, we will use a 5% stress for the restructuring credit event.

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If any of the other soft credit events as discussed in the March 2001 special report (e.g., obligation acceleration without mitigation, bankruptcy without the Successor and Credit Event Supplement) are included in a CDO, we will apply an additional 12.5% stress for each soft credit event. 23

In addition, if physical settlement is an option in the CDO (wherein the counterparty delivers an obligation of a defaulted reference entity in exchange for the full notional amount of the trade), the soft credit events may not be as problematic. Even if a soft credit event occurs, the CDO may receive a performing obligation. To the extent that a CDO can include a waiting period after the maturity date of the credit default swap(s) to account for any performing obligations that may be delivered to the CDO under a soft credit event, a default stress may not be necessary. This would depend on the length of the waiting period and the longest possible deliverable obligation that the CDO could receive. The CDO should be able to hold the obligation until its maturity or until it becomes a "true" (i.e., not soft) defaulted security.

23 These stresses are subject to change should new information come to light during our analysis of the 2003 Definitions or if market conventions should change. Unlike the past in U.S.-based CDOs, we will not offset this stress with an increased recovery rate. The new stresses are intended to account for both the higher default probability and the higher potential recovery rate for soft credit events.

Structural Features Affecting Recovery Rates

Valuation Procedures and Settlement Mechanisms

The procedure and timing for determining the severity of loss on a defaulted reference credit can vary from a bidding procedure that takes place shortly after a default, to reliance on a final work-out value established after the formal work-out process has been completed.

As mentioned, most transactions now allow for the bidding procedure for cash settlement to begin 30 days or more after a credit event occurs. Moody’s assumption on loss severity of a credit event varies depending on the timing in which the bidding process begins. For example, a bond is likely to get a low bid immediately following a credit event because there may not be enough information available for the market to price the bond reasonably. The recovery assumptions used in cash flow CDOs are based on Moody’s historical default studies 24 in which the recoveries for corporate obligations reflect market values 30 days after default. Thus, the cash flow CDO recovery assumptions may serve as the basis for synthetic CDO recovery rates if the bidding procedure begins at least 30 days after the credit event.

Most transactions have also expanded their valuation mechanisms to allow for multiple valuation dates and dealers. In other words, if five dealers are polled in the first round of bidding and no price is determined, the final valuation amount no longer automatically defaults to zero. Generally, the transactions call for the calculation agent to gather additional bids involving different dealers, use different valuation methods (full quotation, weighted average quotation, partial quotation, etc.) and give protection sellers (CDO investors) the ability to provide a bid or to designate a third-party to provide a bid.

Moody’s also requests that a list of eligible dealers be included in the swap documentation or that dealers be limited explicitly to those that are recognized to be major dealers in the debt obligations of the type for which bids are being solicited. Also, these dealers should not be an affiliate of either the reference entity or the protection buyer or of one another.

Synthetic CDOs that can settle physically will receive a deliverable obligation of a defaulted reference entity in exchange for the full notional amount of the trade. The CDO can then dispose of the delivered obligation at an opportune time to maximize recovery. Generally, guidelines in the governing documents require the CDO to dispose of the obligation within a certain time period. Recovery assumptions and the modeling of these recoveries will depend on this timing. For example, if the governing documents allow a CDO to hold the defaulted asset for more than a year, we may lag the recoveries in the model. However, the CDO may be given some credit for the higher expected recoveries.


Default Probability Stress for Restructuring23

<table>
<thead>
<tr>
<th>Stresses</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5%</td>
<td>For unmodified restructuring using the 1999 Definitions</td>
</tr>
<tr>
<td>5.0%</td>
<td>For modified restructuring using the 2003 Definitions or the 1999 Definitions with all Supplements</td>
</tr>
</tbody>
</table>
**Cheapest-to-Deliver Option**

An ongoing Moody's study into recovery values has revealed that there may be certain characteristics of an obligation or trade that create a "cheapest-to-deliver" option in favor of the protection buyer. Even when holding constant the reference entity and the seniority and security of the obligations, there may still be one or more obligations that trade more cheaply than the others. Obviously, protection buyers will seek to use those cheaper credits as valuation obligations (cash settlement) or as deliverable obligations (physical settlement).

Moody's has found that the cheapest-to-deliver option arises through a wide variety of circumstances including (but not limited to) differences in obligation characteristics such as maturity, coupon, covenant package and issuance size as well as non-obligation-related items such as timing of settlement and the credit event that occurred.

Moody's previously identified convertible securities as one example of a cheapest-to-deliver item based on research performed by our fundamental credit analysts. If a convertible security exists, our study on recoveries in synthetic CDOs has shown that in almost all circumstances, it will be the cheapest item to be delivered.

Another cheapest-to-deliver item that was addressed by ISDA in the Restructuring Supplement is a long-dated asset delivered subsequent to a restructuring credit event. To address this concern, ISDA added a Restructuring Maturity Limitation (RML) option to the Definitions. In Europe, the Modified Restructuring Maturity Limitation (mod-RML) option is more commonly used. Both RML and mod-RML ensure that following a restructuring credit event, all eligible deliverable obligations will have final maturities within a "reasonable" time frame. To the extent that neither the RML nor mod-RML options are selected, a cheapest-to-deliver option may be available to the protection buyer at the expense of the protection seller.

Moody's expects to publish several special reports shortly detailing these and other findings from this study. In the meantime, Moody's will assess a 5% cheapest-to-deliver haircut on the recovery rates used for investment grade assets in synthetic CDOs (e.g., modeled recovery rate = cash flow CDO recovery rate * (1-5%)). For non-investment grade assets, the haircut will be 10% (e.g., modeled recovery rate = cash flow CDO recovery rate * (1-10%)), due primarily to the increased likelihood of convertible issuance in the non-investment grade market. Additionally, if neither RML nor mod-RML options are selected, these haircuts will increase to 10% for investment grade assets and 15% for non-investment grade assets. These haircuts are subject to change in the future should new information come to light as a result of our continuing study or if market conditions should change.

**Currency Risk**

Some credit default swaps specify that the settlement currency may be different from the currency under the synthetic CDO (i.e., one or more of the standard specified currencies as defined by ISDA). If the trades are cash settled, the currency risk issue may be mitigated. Whether the obligation used to settle the trade is in Euros or U.S. dollars, for example, the market value may be expressed as a percentage of the notional amount of the trade.

<table>
<thead>
<tr>
<th>Cheapest-to-Deliver Recovery Rate Haircuts</th>
<th>Investment Grade Assets</th>
<th>Non-Investment Grade Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>If no restructuring credit event</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If restructuring credit event with</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>Restructuring Maturity Limitation (RML) or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modified Restructuring Maturity Limitation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mod-RML)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If restructuring credit event without</td>
<td>10%</td>
<td>15%</td>
</tr>
<tr>
<td>Restructuring Maturity Limitation (RML) or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modified Restructuring Maturity Limitation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mod-RML)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Modeled recovery rate = cash flow CDO recovery rate * (1-Haircut)

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26 RML provides that the latest final maturity date for a deliverable obligation is the earlier of 30 months following a restructuring and the latest final maturity date of a restructured bond or loan, provided that this final maturity date is no later than 30 months following the scheduled maturity date of the credit default swap. Mod-RML provides that the latest maturity date for a deliverable obligation is the later of the schedule maturity date of the credit default swap and 60 months following a restructuring for a restructured bond or loan or 30 months following a restructuring for any other deliverable obligation.
If the trade is physically settled, however, the CDO may be left holding a foreign currency obligation whose ultimate recovery value is subject to currency risk. As such, there may need to be an adjustment to the assumed recovery rates to account for the additional volatility. Other alternatives to mitigate this risk include but are not limited to (a) the requirement of cash settlement only in instances where a foreign currency security is delivered, (b) using a fixed exchange rate to shift the currency risk to the counterparty and (c) using a total return swap to mimic the cash flows of the deliverable obligation using a pre-determined exchange rate27.

**SUMMARY**

The most difficult part of rating any CDO is in reconciling the model and documentation with the risks inherent in the structure. Clearly, the relationship between the quantitative and qualitative analyses for synthetic CDOs is especially crucial. Each time we rate a new synthetic CDO, we try to answer the following questions:

- Does the overall modeling approach make sense given the structure of the transaction and the inherent risks that we are trying to capture?
- Does the model portfolio capture all possible permutations of the actual portfolio that may give rise to additional risk?
- Do any of the collateral characteristics lend to different recovery rates than previously assumed?
- Is there anything in the deal’s structure or documentation that may increase the default probability above previously modeled levels?
- Are there any risks not mitigated through documentation that have not or cannot be modeled?

In answering these questions, we have arrived at the ratings approach described in this special report. As synthetic CDO structures continue to mature and transform, market participants (including associations such as ISDA) revise their thinking and more data becomes available, we will continue to refine our own methodologies to meet the challenges ahead.

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27 In option (c), rather than physically receiving the deliverable obligation, it is referenced in a total return swap where the CDO receives any cash flows generated by the obligation at a fixed exchange rate. These cash flows are used to reduce the outstanding balance of the obligation. The collateral manager can choose to sell the obligation at any time at the market price, which is expressed as a percentage of the remaining balance of the obligation or which may also be subject to the fixed exchange rate, depending on the total return swap transaction terms.
APPENDIX A: MOODY’S IDEALIZED DEFAULT PROBABILITY TABLE

<table>
<thead>
<tr>
<th>Moody’s Year</th>
<th>Aaa</th>
<th>Aa1</th>
<th>Aa2</th>
<th>Aa3</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>Baa1</th>
<th>Baa2</th>
<th>Baa3</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>Caa1</th>
<th>Caa2</th>
<th>Caa3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0001%</td>
<td>0.0006%</td>
<td>0.0014%</td>
<td>0.003%</td>
<td>0.058%</td>
<td>0.0109%</td>
<td>0.0389%</td>
<td>0.090%</td>
<td>0.170%</td>
<td>0.420%</td>
<td>0.870%</td>
<td>1.560%</td>
<td>2.810%</td>
<td>4.680%</td>
<td>7.160%</td>
<td>11.620%</td>
</tr>
<tr>
<td>2</td>
<td>0.0002%</td>
<td>0.0010%</td>
<td>0.0080%</td>
<td>0.0190%</td>
<td>0.0370%</td>
<td>0.0700%</td>
<td>0.1500%</td>
<td>0.2800%</td>
<td>0.4700%</td>
<td>1.0500%</td>
<td>2.0200%</td>
<td>3.4700%</td>
<td>5.5100%</td>
<td>8.3800%</td>
<td>11.6100%</td>
<td>16.6100%</td>
</tr>
<tr>
<td>3</td>
<td>0.0007%</td>
<td>0.0100%</td>
<td>0.0260%</td>
<td>0.0590%</td>
<td>0.1170%</td>
<td>0.2220%</td>
<td>0.3600%</td>
<td>0.5600%</td>
<td>0.8300%</td>
<td>1.2000%</td>
<td>2.3800%</td>
<td>3.1300%</td>
<td>4.1800%</td>
<td>5.8000%</td>
<td>9.1300%</td>
<td>15.5000%</td>
</tr>
<tr>
<td>4</td>
<td>0.0018%</td>
<td>0.0210%</td>
<td>0.0470%</td>
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APPENDIX C: ALTERNATIVE DIVERSITY ALLOCATION IN A MULTIPLE BINOMIAL

Author: Gary Witt

In the example described in Table 1, a portfolio of assets was divided into three sub-pools. The overall diversity score for the entire portfolio was 50. In the analysis, diversity scores for the sub-pools were reported as 40, 5 and 5. How were these sub-pool diversities calculated?

Standard Method: Scaling of Pool Diversities to Sum to the Portfolio Diversity

Typically, unadjusted diversity scores (i.e., diversity scores before any scaling is applied) are calculated separately for each sub-pool as if they were unrelated portfolios. These unadjusted diversity scores are then multiplied by a scale factor to ensure that the adjusted sub-pool diversity scores add up to the portfolio diversity score.

In the example described in Table 1, these sub-pool diversity scores were calculated from unadjusted diversity scores of 48, 6 and 6. A scaling factor of $0.833 = \frac{50}{48+6+6}$, the ratio of the overall portfolio diversity to the sum of unadjusted sub-pool diversities, was used to adjust these so that the sum of the adjusted diversity scores would be the overall portfolio diversity of 50.

Scaling down the unadjusted diversity scores is necessary because these unadjusted diversity scores do not account for the correlation across sub-pools. This is unacceptable because the multiple binomial explicitly assumes statistical independence across sub-pools. The simple linear scaling method described above will prevent the diversification from being overstated (and default variation from being understated) in many synthetic CDOs, but it is important to pay attention to the effect that this assumption has on the variation of the default distribution of the idealized multi-pool portfolio around its mean.

This is a brief description of two alternative methods for choosing sub-pool diversities in the multiple binomial. Both methods derive from moment matching: specifically, these two methods preserve the standard deviation of the default distribution implied by the overall pool diversity. If, by applying the standard method, the variance of the pool’s default distribution is reduced, one of the alternatives described below should be considered.

Two formulae are derived. One for the case when all unadjusted sub-pool diversities can be calculated and one for the case when one sub-pool diversity is an unknown and must be implied from the overall diversity and the other sub-pool diversities.

Assumed Distributions

In Moody’s methodology, an idealized portfolio represents the actual portfolio. The expected loss of each liability is then calculated from the default distribution of the idealized portfolio. Here we begin by assuming an idealized portfolio for a multiple binomial of dimension $m$.

Let $X_j$ = the number of defaults in pool $j$ for $j = 1, \ldots, m$.

1) Each $X_j$ is distributed as a $B \left( p_j, D_j \right)$, i.e. a binomial random variable with $D_j$ trials and $E(X_j) = p_j D_j$

2) $X_1, \ldots, X_m$ are independent.

3) The percentage size of each pool $j$ is $w_j$, so that

$$\sum_{j=1}^{m} w_j = 1$$

---

28 Moody’s has different methodologies for calculating diversity score depending on asset type. Whatever method is used, this appendix assumes that the same method will be used to calculate the overall diversity score and all of the unadjusted diversity scores.

29 It is important to note that the overall diversity score explicitly accounts for correlation across sub-pools.

30 For convenience, this method will be referred to in this appendix as the “standard” method.

31 For a multiple binomial there are many possible allocations of sub-pool diversity that preserve the standard deviation of the default distribution. Only two such methods are described here. Other methods may be used in specific transactions.

Moody’s Approach To Rating Synthetic CDOs
The overall portfolio default percentage is $Y$ where

$$Y = \sum_{j=1}^{m} w_j X_j / D_j$$

and the variance of $Y$ is

$$\text{Var}(Y) = \sum_{j=1}^{m} w_j^2 p_j (1 - p_j) / D_j$$

If the number of defaults in this portfolio had been treated as a single binomial $X$ with diversity $D$ and default probability $p$, then the variance of the default percentage would have been

$$\text{Var}(X / D) = p (1 - p) / D$$

The mean of the default distribution under the single binomial assumptions will equal the mean of the default distribution under the multiple binomial assumptions

$$p = \sum_{j=1}^{m} w_j p_j$$

What will be the relationship between the variances of these two distributions?

If the sub-pool diversities are chosen using the standard method described at the start of this appendix, there is no guarantee that the variance under the single binomial assumptions $\text{Var}(X/D)$ will equal the variance under the multiple binomial assumption $\text{Var}(Y)$. This could be problematic. In particular, if $\text{Var}(X/D) > \text{Var}(Y)$ the expected loss of the notes using the multiple binomial may be understated.

**Alternative Method 1: Scaling of Pool Diversities to Match Default Variances**

In this method sub-pool diversities are chosen to achieve two goals: (1) scale all unadjusted diversity calculations by a single scale factor $S$ and (2) preserve the standard deviation of the default distribution implied by the diversity score of the entire portfolio.

If $d_1, \ldots, d_m$ are the unadjusted sub-pool diversity scores and $D$ is the portfolio diversity score, then the variance-scaling factor $S$ is

$$S = \frac{D \sum_{j=1}^{m} w_j^2 p_j (1 - p_j) / d_j}{p (1 - p)}$$

$S$ is used as a multiplier to each unadjusted diversity score $d_j$ and is chosen to insure that

$$\text{Var}(X / D) = \text{Var}(Y)$$

$$D_j = S d_j \quad \text{for } j = 1, \ldots, m$$

are the adjusted sub-pool diversities used to calculate the expected liability losses.
As an example, consider a case where the overall portfolio diversity score is \( D = 25 \) and the unadjusted sub-pool diversities are listed below under the column \( d_j \).

<table>
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<tr>
<th>Sub-pool</th>
<th>( w_j )</th>
<th>( d_j )</th>
<th>( p_j )</th>
<th>( D_j )</th>
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<td>3</td>
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<td>7</td>
<td>5.00%</td>
<td>4.28</td>
</tr>
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</table>

As calculated from the information above, \( S = 0.612 \) was used to calculate the adjusted diversity scores \( D_1, D_2 \) and \( D_3 \). In this case the pool diversities used could be rounded to 8, 8 and 4 as opposed to 10, 10 and 5 using the standard method. In many cases, these two methods yield similar results. However, when \( w_j / D_j \), (the sub-pool portfolio par percentage over the sub-pool diversity) declines with credit quality (i.e. when large \( p_j \Rightarrow \) small \( w_j / D_j \)) the standard method tends to give a higher scale factor, higher sub-pool diversities and hence lower expected losses than a variance scaling method. In general if

\[
S < \frac{D}{\sum_{j=1}^{m} d_j}
\]

then use of alternative method 1 should be considered.

**Alternative Method 2: Solving for One Unknown Sub-Pool Diversity to Match Default Variances**

In some cases it may be useful to solve for one unknown sub-pool diversity to preserve the variance of the overall default distribution. Assume that only one of the unadjusted \( d_j \) is unknown, call it \( d_m \), but all other \( d_j \) are known.

To build a distribution that reflects the actual default distribution of the portfolio, a reasonable assumption is to consider that we should use adjusted \( D_j \), which are equal to unadjusted \( d_j \). This assumption may be particularly appropriate if sub-pool \( m \) is the smallest and/or riskiest of all sub-pools.

\[
D_j = d_j
\]

\[
D_m = \frac{w_m^2 P_m (1 - p_m)}{p(1 - p) / D - \sum_{j=1}^{m-1} w_j^2 p_j (1 - p_j) / d_j}
\]

One specific case where this approach may be useful is in managed CDOs that have covenants for the overall diversity score and for the unadjusted diversity score of some sub-pools. Some static CDOs may also employ from this approach.