Consistent Return Estimates in the Asset Allocation Process - The Black-Litterman Approach

Dr. Werner Koch, CEFA
COMINVEST
PM High Alpha / Portfolio Construction

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The Black-Litterman Approach

Consistent asset return estimates - saving classical mean/variance...

In asset management, the forecast of asset returns is essential within the investment process. In this context, the Black-Litterman approach (1992) yields consistent asset return forecasts as a weighted combination of (strategic) market equilibrium returns and (tactical) subjective forecasts ("views"). The Black-Litterman formalism allows to implement both absolute views (return levels) and relative views (outperforming vs. underperforming assets) for selected assets investigated under „core competence". For any particular view, individual confidence levels for the return estimates have to be specified. The formalism spreads these informations consistently across all assets in the portfolio. The BL-revised returns then serve as a consistent input for mean-variance portfolio optimisation procedures, thus allowing for the implementation of additional constraints. It turns out that BL-optimized portfolios overcome some well-known Markowitz insufficiencies as unrealistic sensitivity to input factors or extreme portfolio weights. The BL process will be introduced both from its theoretical background and its implementation in practice.

Dr. Werner Koch

COMINVEST Asset Management

werner.koch@cominvest-am.com

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On the Added Value of Theories…

About quantitative approaches in finance

Question:

What is the striking difference between option-pricing theory and portfolio theory ?

Answer:

Black-Scholes-Theory is well accepted & established in practice

whereas

Markowitz-Theory is still widely ignored in portfolio management - why ?

Black-Litterman

Major Aspects Considered

- Ways to construct a portfolio
- A straight way: **Markowitz MV**
- Problems with Markowitz in practice
- Enhanced approach: Controlling the inputs via ...

  ... **Black Litterman**, i.e.:
- Combining ...

  ... **Strategic „equilibrium" returns** (as given by theory) with ...

  ... **Tactical „Views" on expected returns** (reflecting core competence)
- yields ...

  ... **consistent BL-expected returns** as inputs for Markowitz MV optimization
- Illustrating **example**: STOXX sector portfolio

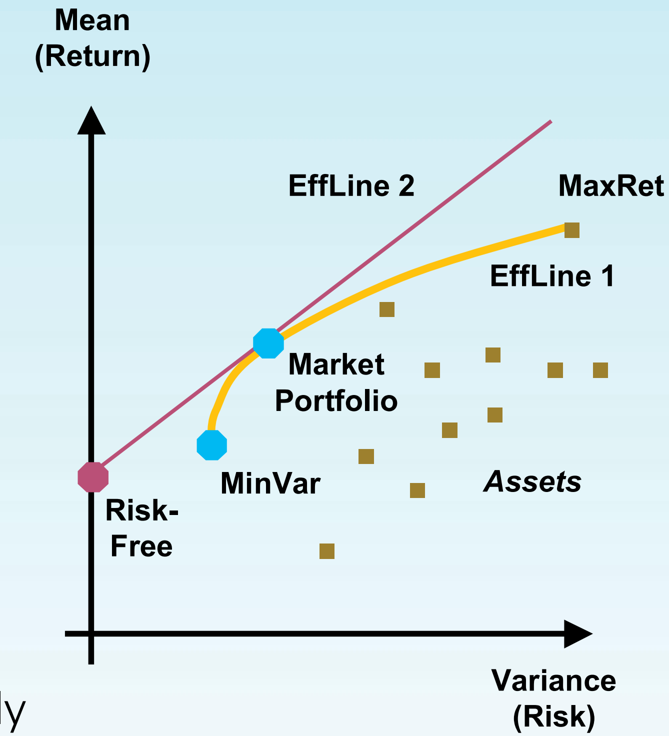
Latest update: October 2004

Data source: DataStream

The Markowitz Approach - in short

Efficient portfolios in the mean-variance approach

- <u>**Starting point** in a world of normally distributed returns</u>: Any asset is described by the first two moments of return - mean and variance.
- In an **efficient portfolio** the assets are weighted such that for a given level of risk a maximum return is achieved (equivalently: for a given return the risk is minimized).
- All efficient portfolios form the **efficiency line** 1. It starts in the minimum variance portfolio and ends in the maximum return portfolio (which is the asset of maximum return).
- If a risk-free asset exists, all efficient portfolios lie on the efficiency line 2, starting at the risk-free asset and tangentially touching efficiency line 1. Efficient portfolios are a combination of the risky **market portfolio and the risk-free asset** (risk-free *long* or *short*), a.k.a. *Tobin Separation theorem (1958)*.



The Markowitz Approach - dealing with its problems

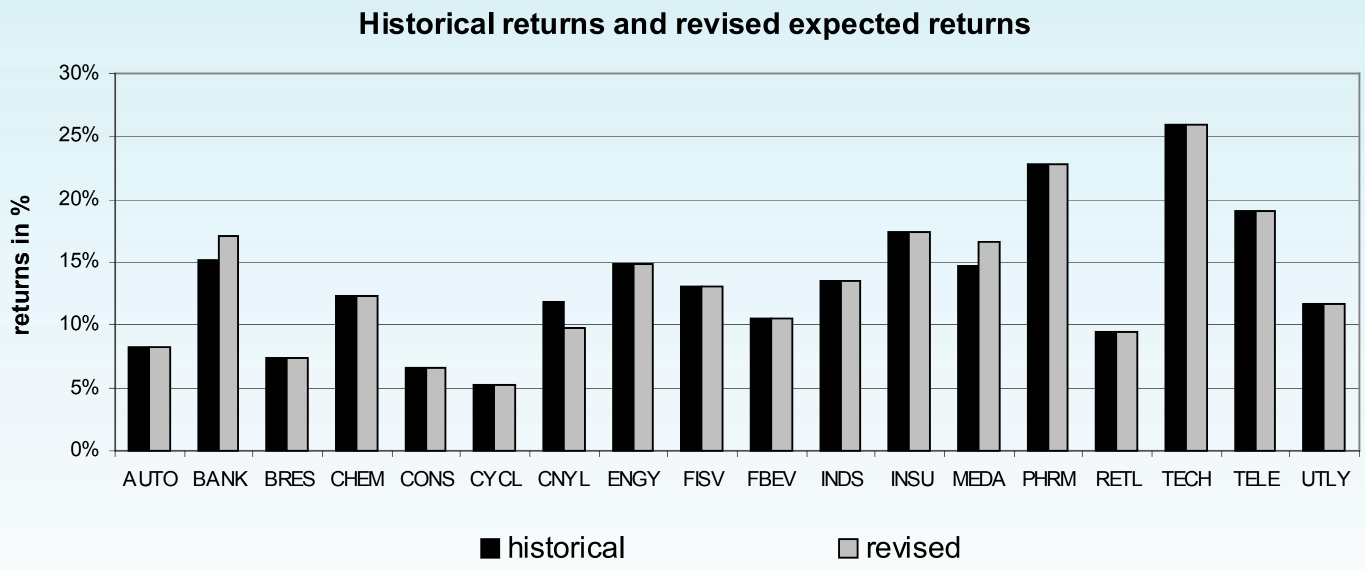
Deficits of the mean-variance (MV) concept, suggestions for solutions...

Deficit	Solution
High sensitivity on inputs (return estimates!) leads to large weight fluctuations in the optimal portfolio.	***Black-Litterman***
„Corner solutions“: Extreme portfolio weights (also in the case of optimization algorithms using constraints)	***Black-Litterman***
Aggregation: Consistent aggregation of huge number of estimated returns overburdens the investment process	***Black-Litterman***
No quantification of confidence in estimated returns	***Black-Litterman***
One-periodical approach	Multi-period approaches, ...
„Variance“= restricting risk to symmetric return volatility	Extending to VaR, ...
Requires ex-ante-estimates of covariance matrix	Vola-modeling, ...

The Markowitz Approach - straight is not enough

When return estimates change...

- Let the investment universe be the 18 STOXX sectors in Euroland.
- At time 0: Returns : Historical returns
  Weights : via MV
- At time 1: Returns : +2%pts for BANK and MEDA, -2%pts for CNYL, others unchanged
  Weights : via MV

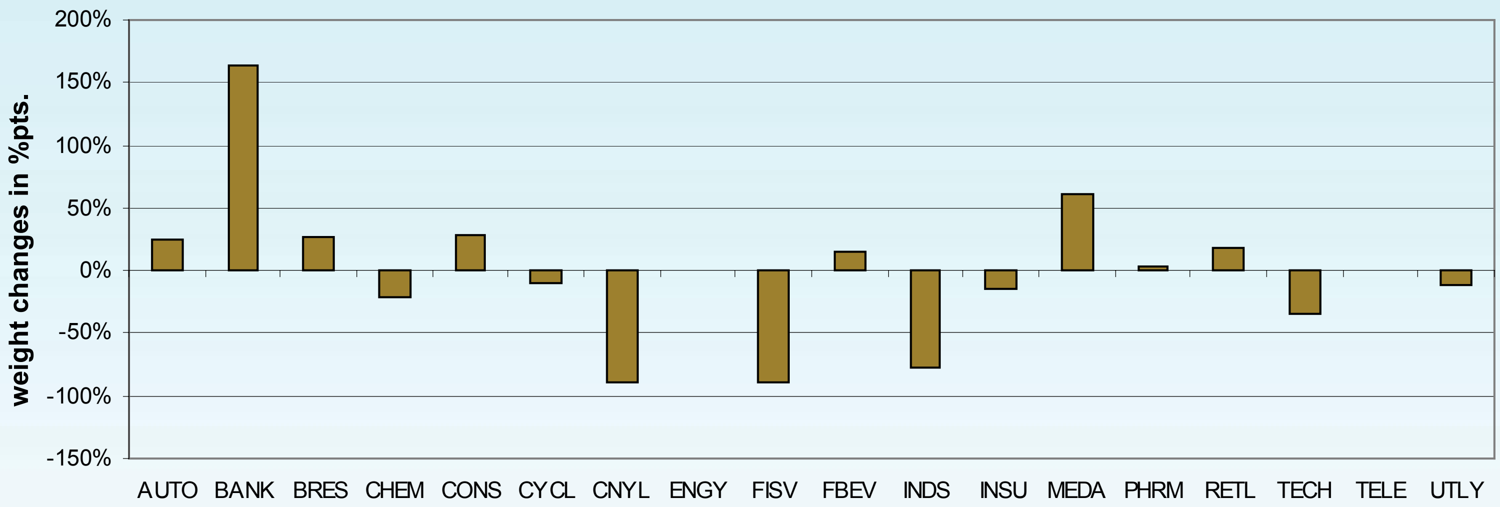


The Markowitz Approach - straight is not enough

Fluctuation of asset weights with changed return estimates

- Even small and selected changes in expected returns lead to huge unrealistic shifts in asset weights! (γ=3, historical covariances)

Changes in portfolio weights due to changes of expected returns



- Problems: Communication of results, (re-) allocation in real portfolios, acceptance of method ?!

The Markowitz Approach - straight

The formal optimization approach for risky assets, basic outline.

- Markowitz Theory relates risk & return
- MV optimization problem:

$$w^{T} R - \frac{\gamma}{2} \cdot w^{T} \Omega\, w \rightarrow \max_{w}$$

R	=	vector of returns
Ω	=	covariance matrix
γ	=	risk aversion parameter
w	=	vector of weights

- Solution: Optimal portfolio weights w^* (no constraints):

$$w^{*} = (\gamma\, \Omega)^{-1} R$$

- Markowitz provides a mechanism to achieve optimal (efficient) portfolios.
  What about the input factors ???

Extending the Markowitz Approach

Equilibrium returns

Supply & demand

- Traditional approach of maximum return & minimum risk is demand-side perspective.
- Need to balance with supply-side...

Concept of equilibrium returns:

- The market portfolio exists in market equilibrium, i.e. supply & demand are in equilibrium.
- Therefore, equilibrium returns reflect neutral „fair“ reference returns Π:
- Inverse optimization yields:

$$\Pi = (\gamma\,\Omega)\,w_{MCap}$$

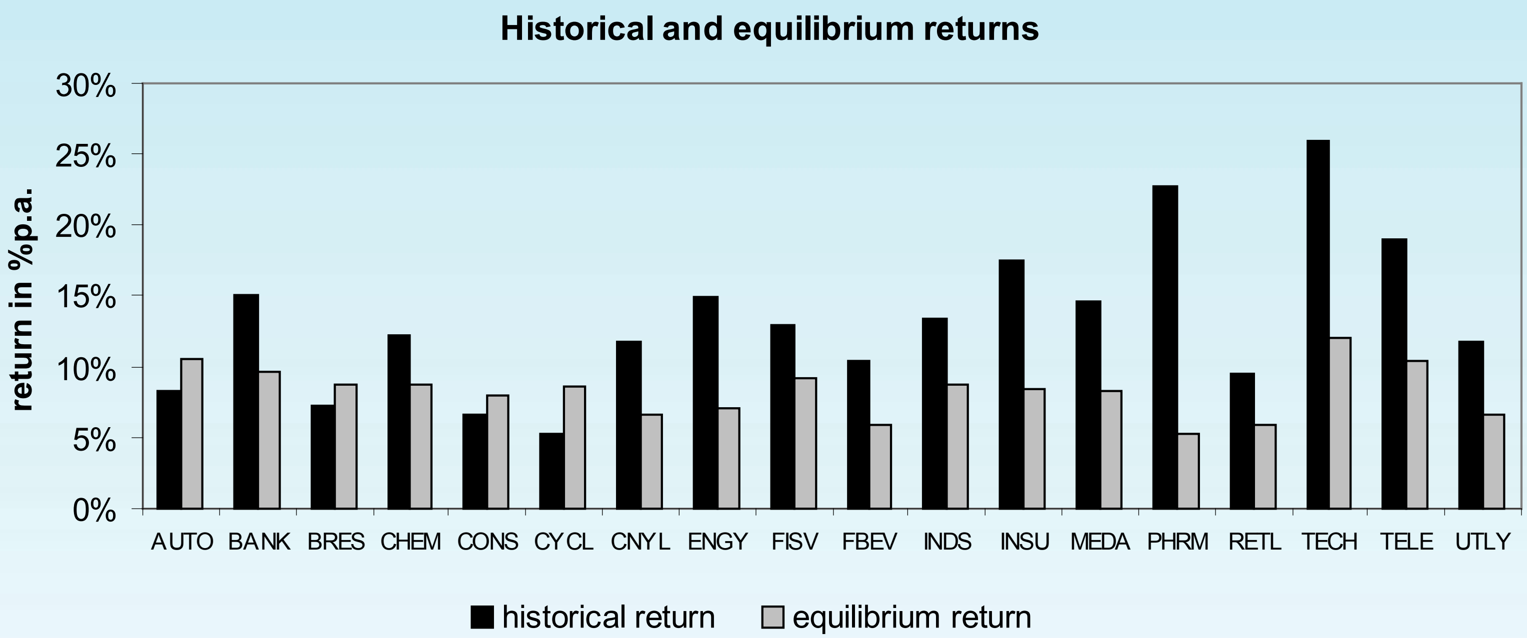
w_{MCap} = vector of assets‘ market capitalization

Conclusion

- Use of equilibrium returns as a long term strategic reference for any return estimate („market neutral starting point“).

Extending the Markowitz Approach

Equilibrium (or implicit) returns for the STOXX sectors



- These implicit returns serve as reference returns for all investigations („market neutral starting point “)
- Note that equilibrium returns are *calculated*; they do not require any estimation procedure.

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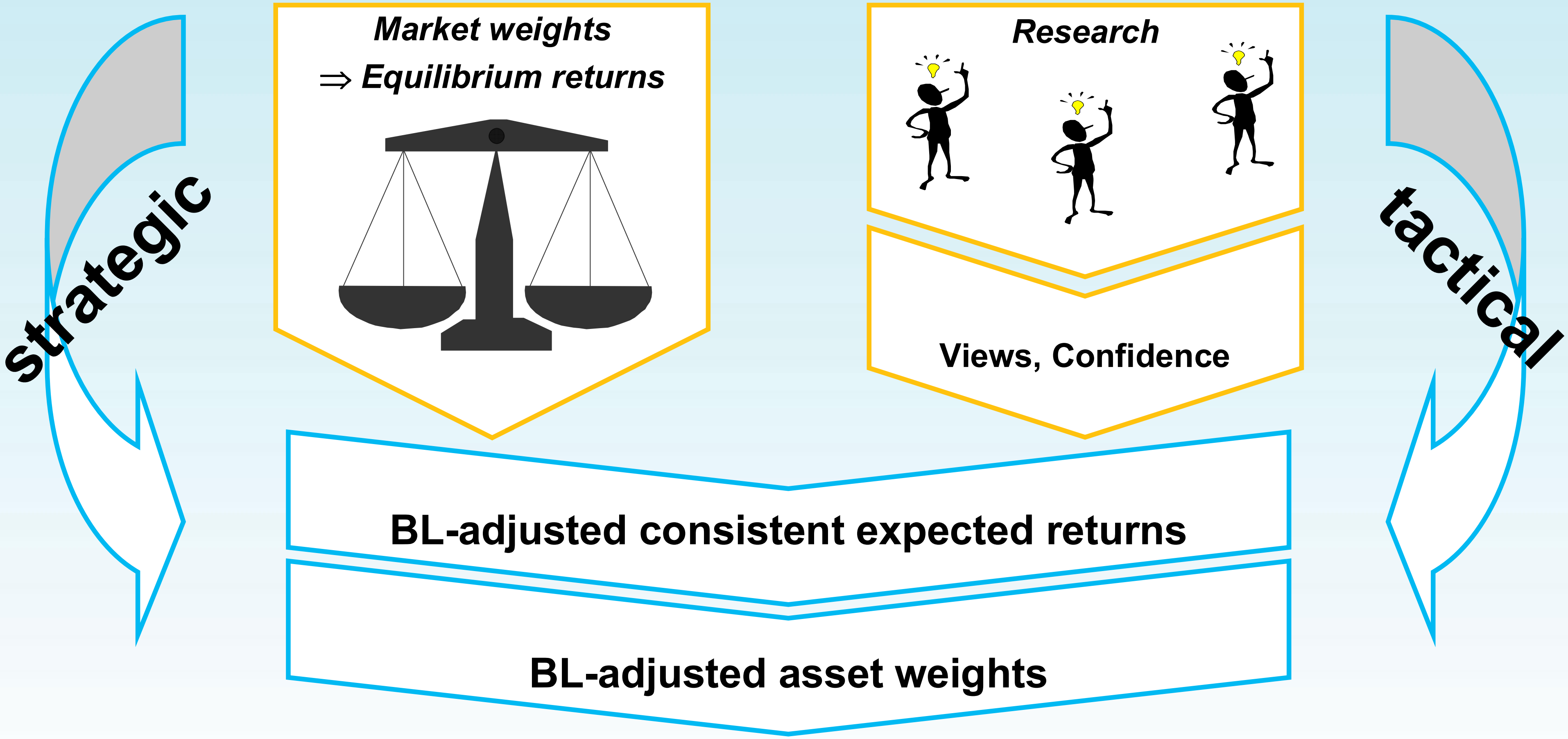
Black-Litterman Approach - basic outline

BL-Prozess provides a mechanism to combine long term equilibrium returns with short term return estimates as a basis for tactical positioning.



Black-Litterman Approach - going math

Optimization s.t. contraints

- Determine the optimal estimate E(R), which minimizes the variance of E(R) w.r.t. equilibrium returns Π: $E(R) = \Pi + \nu \quad \text{with} \quad \nu \sim N(0, \tau\,\Omega).$

$$[E(R) - \Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E(R) - \Pi] \rightarrow \min_{E(R)}$$

s.t.

$$P \cdot E(R) = \begin{cases} V & \text{certain Views} \\ V + e & \text{uncertain Views} \end{cases}$$

where: $P \cdot E(R) \sim N(V, \Sigma)$, $\Sigma_{ii} = e_i$

COMINVEST ASSET MANAGEMENT

Black-Litterman Approach - the formulas

Equations for optimal BL-return estimates

- Solution in the case of **certain estimates** ($\Sigma \equiv$ zero matrix):

$$\overline{E}(R) = \Pi + (\tau\,\Omega)\,P^{\mathrm{T}} \cdot \left(P(\tau\,\Omega)\,P^{\mathrm{T}}\right)^{-1} \cdot \left(V - P\,\Pi\right)$$

- Solution in the case of **uncertain estimates** (Σ = diagonal matrix):

$$\overline{E}(R) = \left[(\tau\,\Omega)^{-1} + P^{\mathrm{T}}\Sigma^{-1}P\right]^{-1} \cdot \left[(\tau\,\Omega)^{-1}\Pi + P^{\mathrm{T}}\Sigma^{-1}V\right]$$

- The **constraints** $P \cdot E(R) = V$ are implicitly fulfilled.

Black-Litterman Approach - more math I

Formal proof for the case „certain estimates“

Proposition: The optimization problem $[E(R)-\Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E(R)-\Pi] \to \min\limits_{E(R)}$ s.t. $P \cdot E(R) = V$

yields variance-minimum returns $\overline{E}(R) = \Pi + (\tau\,\Omega)\,P^{\mathrm{T}} \cdot \left(P(\tau\,\Omega)\,P^{\mathrm{T}}\right)^{-1} \cdot (V - P\,\Pi)$

Proof:

Lagrangian $\quad L := [E-\Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E-\Pi] - \lambda \cdot (PE - V)$

f.o.c.'s: $\quad (1)\ \dfrac{\partial L}{\partial E} = 0 \quad \text{and} \quad (2)\ \dfrac{\partial L}{\partial \lambda} = 0$

"scalarizing" $\quad \dfrac{\partial L}{\partial E_i} = \dfrac{\partial}{\partial E_i}\left\{\tau^{-1}\sum_{j,k} E_j \Omega^{-1}_{jk} E_k - \sum_k \lambda_k \left(\sum_j P_{kj} E_j - V_k\right)\right\} = 2\tau^{-1}\sum_k \Omega^{-1}_{ik} E_k - \sum_k P_{ki}\lambda_k$

$$\frac{\partial L}{\partial \lambda_i} = -\frac{\partial}{\partial \lambda_i}\sum_k \lambda_k \left(\sum_j P_{kj} E_j - V_k\right) = -\left(\sum_j P_{ij} E_j - V_i\right)$$

"vectorizing": $\quad (1)\ \dfrac{\partial L}{\partial E} = 2(\tau\,\Omega)^{-1}\,E - 2(\tau\,\Omega)^{-1}\,\Pi - P\lambda = 0 \quad \text{and} \quad (2)\ \dfrac{\partial L}{\partial \lambda} = PE - V = 0$

Solve Eq. (1) for E, insert in Eq.(2), thereof expression for λ, result follows with Eq. (1). ◊

Black-Litterman Approach - more math II

Formal proof for the case „uncertain estimates“ (BL-Master Formula)

Proposition: $[E(R)-\Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E(R)-\Pi] \rightarrow \min\limits_{E(R)}$ with constraints $P \cdot E(R) = V + e$

yields variance-minimum returns $\bar{E}(R) = \left[(\tau\,\Omega)^{-1} + P^{\mathrm{T}}\Sigma^{-1}P\right]^{-1} \cdot \left[(\tau\,\Omega)^{-1}\Pi + P^{\mathrm{T}}\Sigma^{-1}V\right]$

Proof:

Given: $\Pi = E(R) + v$ and $V = P \cdot E(R) + e$

Setting $Y := \begin{pmatrix} \Pi \\ V \end{pmatrix}$, $X := \begin{pmatrix} I \\ P^T \end{pmatrix}$, $W := \begin{pmatrix} \tau\Omega & 0 \\ 0 & \Sigma \end{pmatrix}$ and $u \sim N(0, W)$

so that $Y = X \cdot E(R) + u$ and using generalized least square $E(R) = \left(X^T W^{-1} X\right)^{-1} X^T W^{-1} Y$ we get

$$E(R) = \left[\begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau\Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix}\right]^{-1} \times \left[\begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau\Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} \Pi \\ V \end{pmatrix}\right]$$

$$= \left[\begin{pmatrix} (\tau\Omega)^{-1} & P^T\Sigma^{-1} \end{pmatrix} \begin{pmatrix} I \\ P \end{pmatrix}\right]^{-1} \times \left[\begin{pmatrix} (\tau\Omega)^{-1} & P^T\Sigma^{-1} \end{pmatrix} \begin{pmatrix} \Pi \\ V \end{pmatrix}\right] = \left[(\tau\Omega)^{-1} + P^T\Sigma^{-1}P\right]^{-1} \times \left[(\tau\Omega)^{-1}\Pi + P^T\Sigma^{-1}V\right] \lozenge$$

Elegant proof, taken from *„Asset Allocation Model“, Daniel Blamont, Global Markets Research, Dt.Bank, July 30 2003*

Black-Littermann - the master formula

$$E(R) = \left[(\tau\,\Omega)^{-1} + P^{T}\Sigma^{-1}P\right]^{-1} \cdot \left[(\tau\,\Omega)^{-1} \cdot \Pi + P^{T}\Sigma^{-1}\underbrace{P \cdot P^{-1}}_{=1} V\right]$$

- Complex interdependencies between equilibrium returns and subjective return expectations
- First factor (role of „Denominator“) : Normalisation
- Second factor (role of „Numerator“) : Balance between returns Π (= equilibrium returns) and V (= Views). Covariance $(\tau\,\Omega)^{-1}$ and confidence $P^{T}\Sigma^{-1}P$ serve as weighting factors.
- With: Matrix $\tau\,\Omega$: covariance of historical returns, τ = parameter
  Matrix P : formal aggregation of Views
  Matrix Σ^{-1} : confidence in Views (Σ : „covariance of estimated Views“)
  Σ assumed to be diagonal, i.e. no cross-informations on Views.
- Limiting case 1: no estimates $\Leftrightarrow$ P=0: $E(R) = \Pi$ i.e. BL-returns = equil. returns.
- Limiting case 2: no estimation errors $\Leftrightarrow$ $\Sigma^{-1} \to \infty$: $E(R) = P^{-1}V$ i.e. BL-returns = View returns.

Black-Litterman Approach - remarks

Use of CAPM to determine equilibrium returns

- Instead of inverse optimization: Equilibrium returns Π_{Eq} from CAPM
- Additional input for CAPM: Risk-free rate r_f, risk premium vs market (M), Beta coefficients
- Evaluation:
  (fair return for asset i)
  $$\Pi_{i,Eq} = r_f + \beta_i \cdot (r_M - r_f) \quad \text{with} \quad \beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

BL returns as a Bayesian a-posteriori estimator

- Bayes Theorem (or "Law" or "Rule") states how to determine conditional expectations.
- Given an a-priori known distribution of a random variable. Adding new information leads to a revised conditional distribution, the so-called a-posteriori distribution („learning").
  BL-return estimates are a-posteriori (multivariate) normally distributed return expectations.

Black-Litterman Approach - γ

Remark: Risk aversion parameter γ

- How does risk change for a one bp higher return?
- **Suggestions:**

Satchell & Scowcroft and Best & Grauer:

Let $\gamma = (r_M - r_f) / \sigma_M^2$

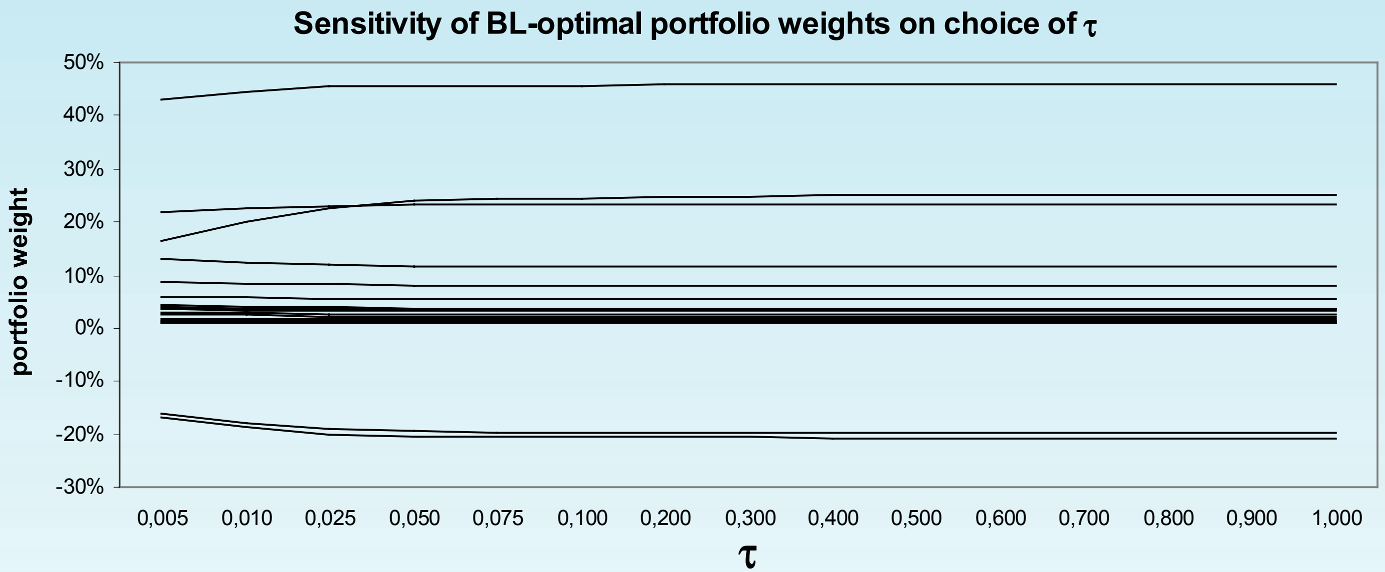
where $\sigma_M^2 = w^T \, \Omega \, w$, w = market cap.

Zimmermann et al.: σ_M=16.9% p.a. for STOXX-data (own calculation)
Let $\gamma = 3$, which corresponds to a risk premium of 8.6%.

Idzorek: (DJIA, USA): Risk premium=7.5%: $\gamma = 2.25$.

Black-Litterman Approach - τ

Remark: Skaling parameter τ for the covariance matrix



Results/Setting

- Covariances of expected returns proportional to historical covariances: $\tau\ \Omega$.
- τ measures confidence in benchmark, i.e. overall balance between BM and Views.
- τ small: *VAR[E(R)] << VAR[historical returns]*
- $\tau = 0.3$ "plausible" (used for numerical evaluations throughout).

Black-Litterman Approach - some real problems

Additional remarks on the recent remarks

- Calibration problems with parameter τ ("plausible", "adjusted to IR=1", ...)
- Calibration problems with parameter γ ("world wide risk aversion", ...)
- Calibration problems with expressing the degree of confidence ("1..3", "0..100%")

Black-Litterman Approach - Views

Implementing Views on expected returns deviating from equilibrium figures

Views

- Return estimates differing from the (strategic) equilibrium returns are the essential input to the BL estimation process.

Specification of Views

- ... as absolute return expectations for individual assets

  and / or

- ... as relative return expectations relating a number of assets or aggregates of assets.

  Formal constraint: #Views $\leq$ #Assets.

Confidence

- Each View has to be assigned the level of confidence for an interval of uncertainty.

Selective Views

- Views can be restricted to selected assets for which in-depth analysis is available.

Black-Litterman Approach - Views

- A **relative View** can be formulated as follows: „The sectors Pharmacy and Industry will outperform Telecom and Technology by 3% ± 1% with a confidence of 90%“:

$$\left[w_{PHRM} \cdot E(R_{PHRM}) + w_{INDU} \cdot E(R_{INDU})\right] - \left[w_{TELE} \cdot E(R_{TELE}) + w_{TECH} \cdot E(R_{TECH})\right] = 3\% + (0.61\%)^2$$

- Basically: A *long*-portfolio with outperformers, a *short*-portfolio with underperformers.

- An **absolute View** can be formulated as follows: „The sector of Non-Cyclical Goods (CNYL) will perform better than stated by the equilibrium return of 6.66%. Our target return is 7.5% with 90% of confidence within a range of ±1.5%“:

$$1 \cdot E(R_{CNYL}) = 7.5\% + (0.91\%)^2$$

Black-Litterman Approach - combining Views I

Formal aggregation of Views

- Relative and absolute Views form a system of linear equations as a constraint to the optimization problem:

$$P \cdot E(R) = V + e$$

where (k = #Views and n = #Assets, with k ≤ n):

E(R)	= n×1 vector of expected asset returns, unknown
P	= k×n matrix, representing the Views
V	= k×1 vector, absolute / relative return expectations (i.e., levels or over-/underperforming)
e	= k×1 vector of squared StDev's

(note that Σ^{-1} is a k×k diagonal matrix expressing confidence (assuming independent estimation errors) with $\Sigma_{ii} = e_i$)

Black-Litterman Approach - combining Views II

Result shown from the example following on page 27.

- Combining the aforementioned Views using $P \cdot E = V + e$, we get:

$$P \cdot \begin{pmatrix} E(R_{AUTO}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = \begin{pmatrix} 3\% \\ 7.5\% \end{pmatrix} + \begin{pmatrix} (0.61\%)^2 \\ (0.91\%)^2 \end{pmatrix}$$

- where

long positions | *short* positions

$$P = \begin{pmatrix} View\,1, rel. \\ View\,2, abs. \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 & \cdots & 0 & 0.34 & 0 & 0 & 0.66 & 0 & -0.51 & -0.49 & 0 \\ 0 & \cdots & 1 & 0 & & \cdots & & & \cdots & & & \cdots & 0 \end{pmatrix}$$

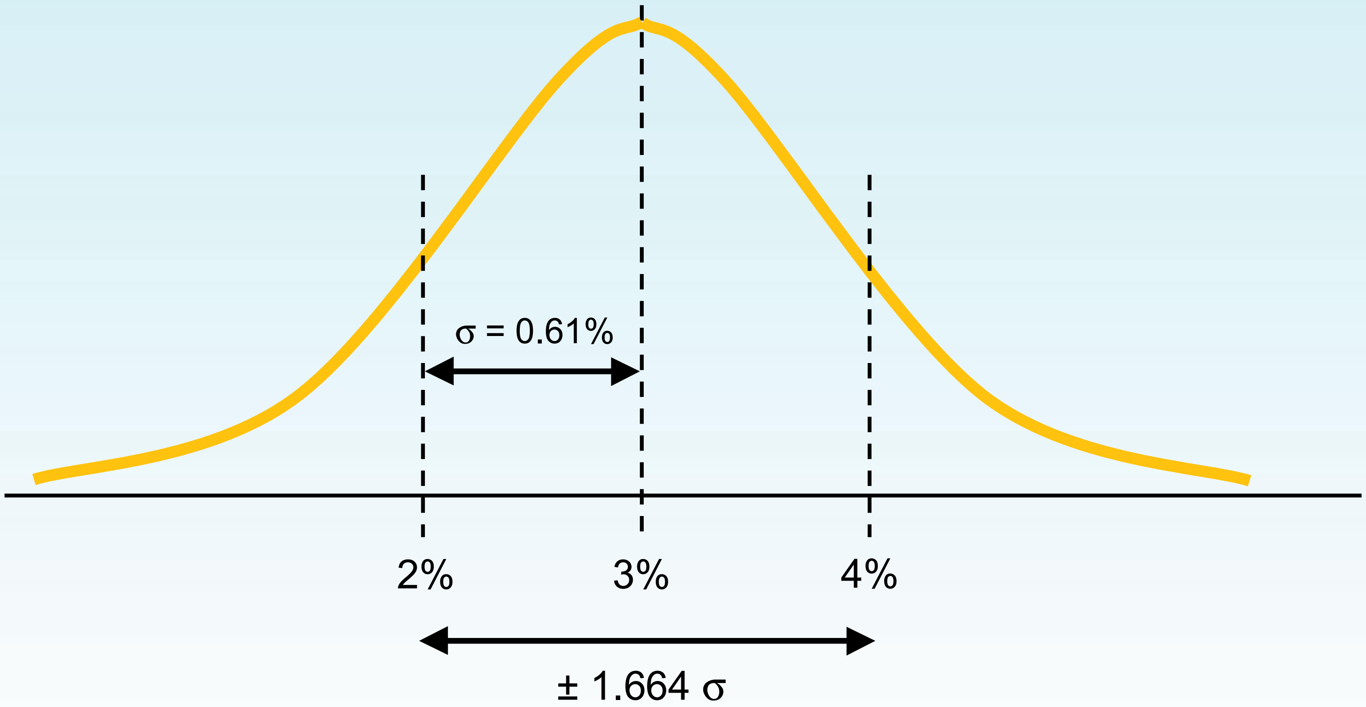
AUTO ⋮ CNYL ENGY FISV FBEV INDS INSU MEDA PHRM RETL TECH TELE UTLY

asset projector

Black-Litterman Approach - confidence

Technical note on „confidence“

- Comment on determination of *e* : The fact that the amount of, e.g., *relative* outperformance (View 1) of 3% ±1% is assigned a 90% probability is interpreted within a normal distribution.
- ***mean*** **=** μ **= 3%** and ***variance*** **=** ***VAR*** **=** σ^2 **= (0.61%)² =** $e_1 \equiv \Sigma_{11}$.



Black-Litterman Approach - example in detail

„Sector allocation, Dow Jones STOXX“

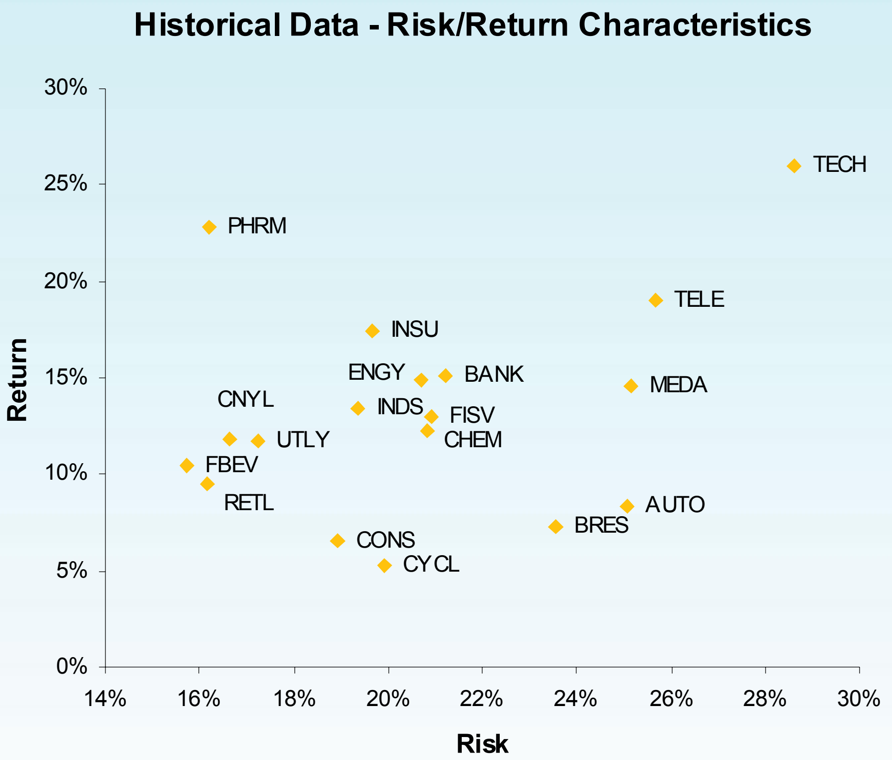
- Example follows the lines of
  „Einsatz des Black-Litterman-Verfahrens in der Asset Allocation“, *H.Zimmermann et al.*
  publ. in „Handbuch Asset Allocation“
  (Editors: Dichtl, Schlenger u. Kleeberg, publ. by Uhlenbruch-Verlag, 2002).
- Notation, scenarios and data therein have been used, some data are missing.
- Missing data - volatilities and covariances - had to be calculated, causing some deviations in the numerical results between this presentation and the reference .
  Nevertheless, all relevant results are reproduced.
- All calculations can be / have been implemented and performed in Excel (TM).

Black-Litterman Approach - the data

Sectors in the Dow Jones STOXX index

- Monthly returns in Sfr (Swiss francs), period: 06/1993 - 11/2000, annualized data.

Sector	hist.Return	hist.Volatility	MarketCap
total: 18	average: 16,22%		total: 100,01%
AUTO	8,32%	25,09%	1,65%
BANK	15,14%	21,21%	15,04%
BRES	7,31%	23,56%	1,22%
CHEM	12,25%	20,81%	1,80%
CONS	6,56%	18,92%	1,26%
CYCL	5,24%	19,94%	2,85%
CNYL	11,80%	16,66%	2,90%
ENGY	14,92%	20,72%	10,30%
FISV	13,01%	20,91%	4,12%
FBEV	10,47%	15,72%	4,59%
INDS	13,45%	19,35%	5,19%
INSU	17,43%	19,68%	6,89%
MEDA	14,63%	25,17%	3,27%
PHRM	22,83%	16,20%	10,24%
RETL	9,49%	16,16%	2,27%
TECH	25,95%	28,60%	11,03%
TELE	18,99%	25,69%	10,56%
UTLY	11,77%	17,25%	4,83%



Black-Litterman Approach - the correlations

Correlation matrix of Dow Jones STOXX sectors

- Calculation based on monthly returns

	AUTO	BANK	BRES	CHEM	CONS	CYCL	CNYL	ENGY	FISV	FBEV	INDS	INSU	MEDA	PHRM	RETL	TECH	TELE	UTLY
AUTO	**100%**	**74%**	**73%**	**83%**	**78%**	**75%**	**73%**	55%	**73%**	**71%**	**79%**	**72%**	46%	43%	68%	69%	65%	64%
BANK	**74%**	**100%**	63%	**73%**	**74%**	**75%**	**71%**	59%	**92%**	**75%**	**75%**	**87%**	39%	63%	62%	67%	59%	64%
BRES	**73%**	63%	**100%**	**83%**	**81%**	**78%**	60%	66%	69%	56%	**78%**	56%	44%	31%	59%	62%	52%	41%
CHEM	**83%**	**73%**	**83%**	**100%**	**85%**	**82%**	**72%**	67%	**72%**	**74%**	**82%**	70%	51%	45%	69%	65%	57%	57%
CONS	**78%**	**74%**	**81%**	**85%**	**100%**	**90%**	**72%**	66%	**75%**	**76%**	**89%**	64%	54%	39%	67%	66%	63%	64%
CYCL	**75%**	**75%**	**78%**	**82%**	**90%**	**100%**	67%	64%	**79%**	70%	**87%**	63%	58%	43%	67%	**70%**	63%	56%
CNYL	**73%**	**71%**	60%	**72%**	**72%**	67%	**100%**	55%	69%	**75%**	69%	**74%**	41%	58%	**73%**	53%	59%	**71%**
ENGY	55%	59%	66%	67%	66%	64%	55%	**100%**	59%	59%	59%	54%	28%	43%	55%	43%	32%	46%
FISV	**73%**	**92%**	69%	**72%**	**75%**	**79%**	69%	59%	**100%**	**75%**	**73%**	**85%**	39%	61%	58%	64%	57%	58%
FBEV	**71%**	**75%**	56%	**74%**	**76%**	70%	**75%**	59%	**75%**	**100%**	62%	**74%**	27%	63%	61%	40%	41%	66%
INDS	**79%**	**75%**	**78%**	**82%**	**89%**	**87%**	69%	59%	**73%**	62%	**100%**	65%	**72%**	38%	68%	**82%**	**77%**	67%
INSU	**72%**	**87%**	56%	70%	64%	63%	**74%**	54%	**85%**	**74%**	65%	**100%**	36%	67%	61%	60%	56%	68%
MEDA	46%	39%	44%	51%	54%	58%	41%	28%	39%	27%	**72%**	36%	**100%**	21%	42%	**75%**	**77%**	57%
PHRM	43%	63%	31%	45%	39%	43%	58%	43%	61%	63%	38%	67%	21%	**100%**	43%	35%	37%	58%
RETL	68%	62%	59%	69%	67%	67%	**73%**	55%	58%	61%	68%	61%	42%	43%	**100%**	52%	53%	57%
TECH	69%	67%	62%	65%	66%	**70%**	53%	43%	64%	40%	**82%**	60%	**75%**	35%	52%	**100%**	**81%**	55%
TELE	65%	59%	52%	57%	63%	63%	59%	32%	57%	41%	**77%**	56%	**77%**	37%	53%	**81%**	**100%**	70%
UTLY	64%	64%	41%	57%	64%	56%	**71%**	46%	58%	66%	67%	68%	57%	58%	57%	55%	70%	**100%**

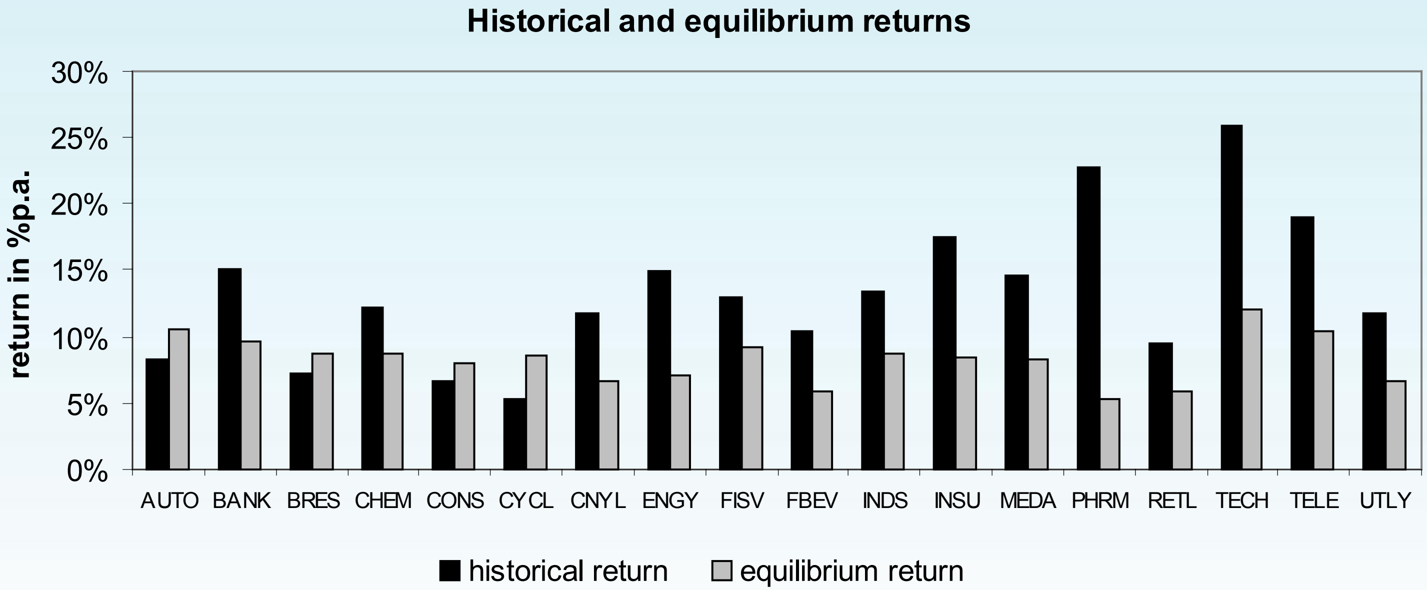
- Covariance matrix via $\Omega_{ij} = \sigma_i \, \sigma_j \, \rho_{ij}$.

Black-Litterman Approach - the equilibrium returns

BL-starting point: Equilibrium - implicit - returns of market portfolio

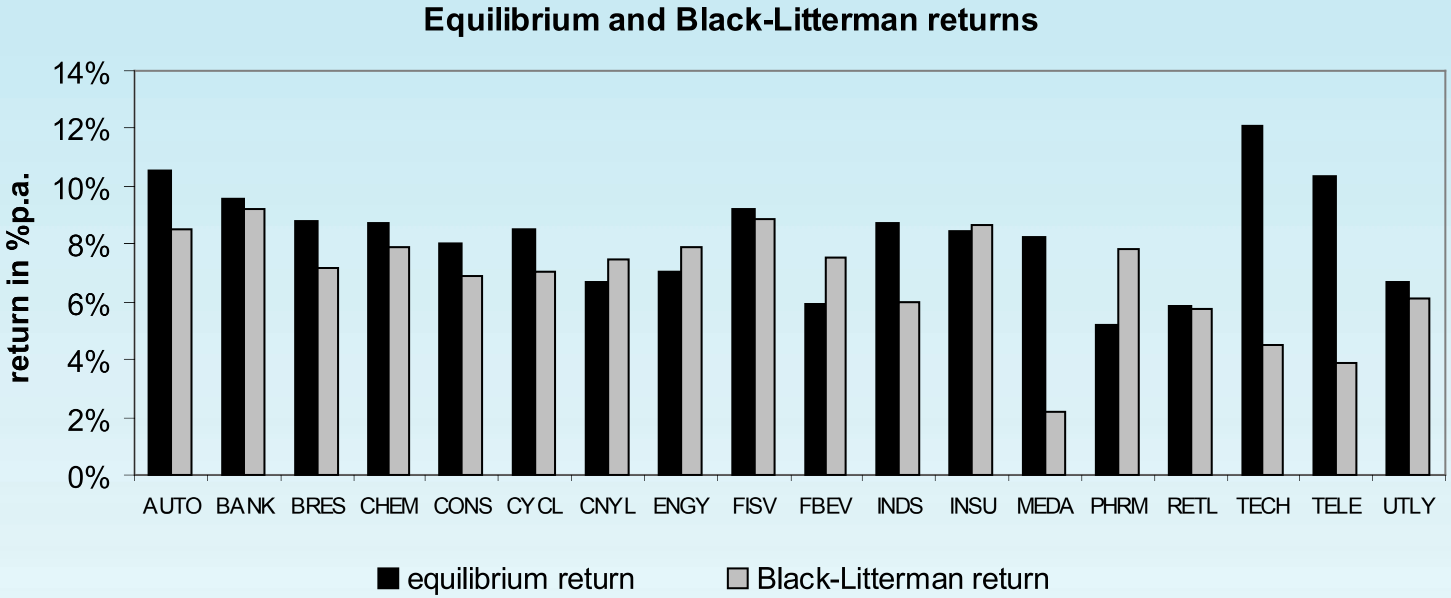
- "inverse optimization" yields **equilibrium returns**

$$R_{Equilibrium} = (\tau\,\Omega)\cdot w_{Equilibrium} \qquad \text{where} \quad w_{Equilibrium} = w_{Market\ Portfolio}$$



Black-Litterman Approach - the BL returns

From equilibrium returns to Black-Litterman returns (Views as given)

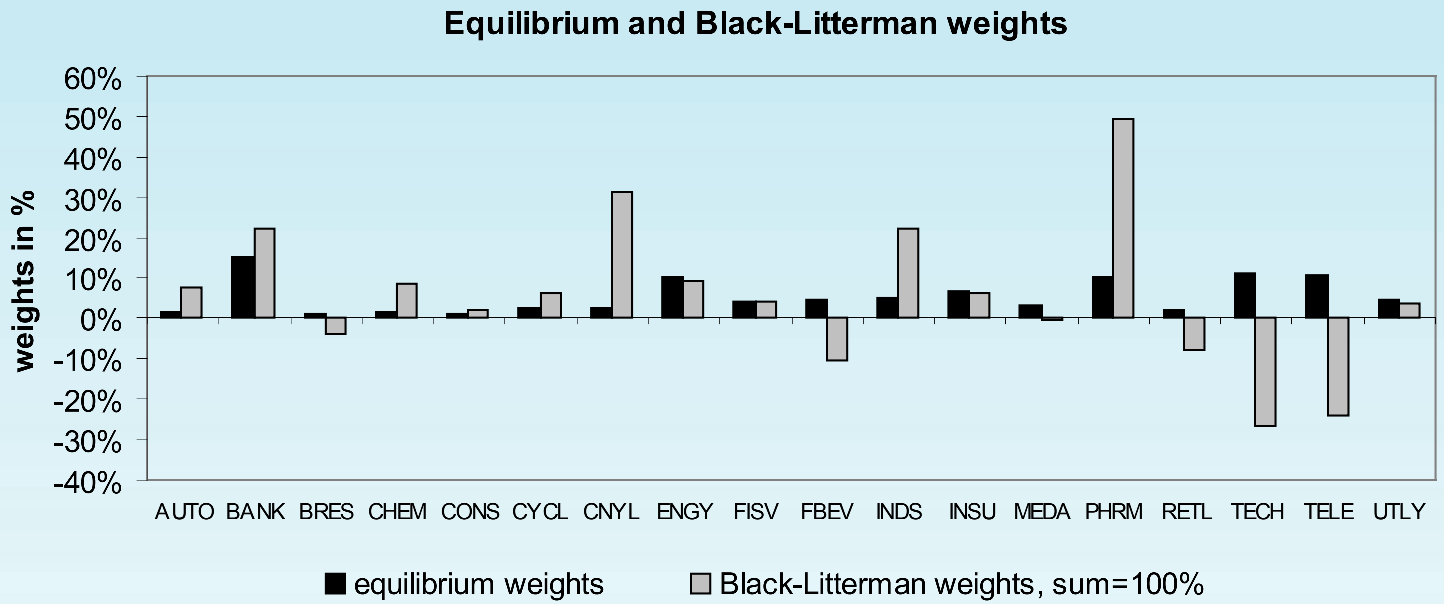


Having implemented the Views... :

- BL return expectations significantly lowered for TECH und TELE.
- BL return expectation higher in PHRM but lower in INDS (ok, because the *relative* View „... better than TELE und TECH" remains intact !)
- For CNYL, the expected return shifts from 6.66% to 7.48% (s.t. 90% confidence for 7,5%).
- Example: MEDA (correlated by 75% to TECH, 77% to TELE) has significantly lower return.

Black-Litterman Approach - the BL weights

Comparing equilibrium weights (market cap.) and Black-Litterman weights

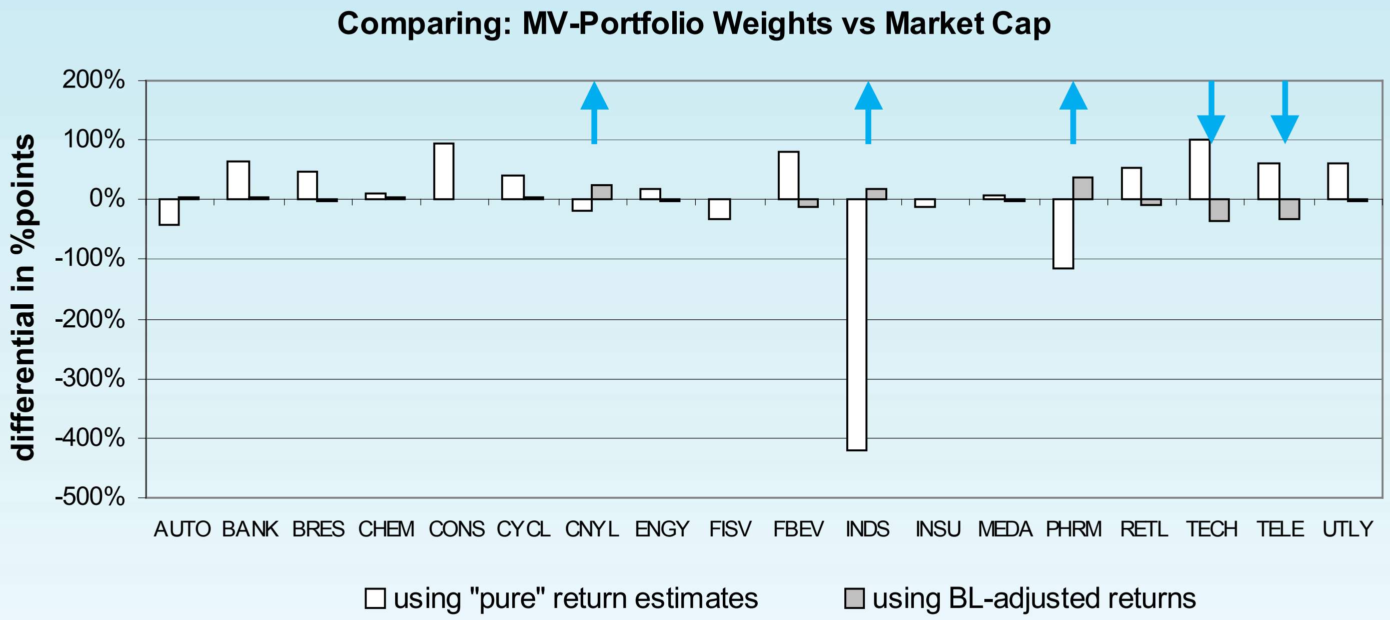


Using the BL returns (note: strong confidence in Views of 90%):

- Significant <u>weight reduction</u> for TELE and TECH (also, e.g., for MEDA)
- Significant <u>weight increase</u> for INDS and PHRM
- Significantly <u>increased weight</u> for CNYL due to higher return expectation (+0.84%pts)

Black-Litterman Approach vs straight MV

Given the return scenario („Views“,↑↓), the revised portfolio clearly benefits from the BL-enhanced optimization process.



- **Straight MV optimization** - which is a naiv approach in terms of „c.p.“ return estimates - yields extreme and unreliable changes in portfolio weights
- **The BL-adjusted return input for MV** stabilizes the weights, leading to a reliable new portfolio structure.

COMINVEST ASSET MANAGEMENT

Black-Litterman Approach - constraints I

Calculation of weights s.t. constraints

- **In general: Use mean/variance optimizer with constraints**
- **No constraints:**

$$w_{BL} = \frac{1}{\gamma} \Omega^{-1} \cdot R_{BL}$$

- **"Budget constraint", i.e. sum of weights = 100%** ($I = 1\text{-}Vector$)**:**

$$w_{BL} = \frac{\Omega^{-1} I}{I^{T} \Omega^{-1} I} + \frac{1}{\gamma} \Omega^{-1} \cdot \left(R_{BL} - \frac{I^{T} \Omega^{-1} R_{BL}}{I^{T} \Omega^{-1} I} I \right)$$

Black-Litterman Approach - constraints II

Calculation of weights s.t. constraints

- Additional constraint: **Tracking Error**

$$TE^2 = \sigma_P^2 + \sigma_M^2 - 2 \cdot Cov(R_P, R_M)$$

- Additional constraint: **Portfolio-BETA** („directional risk in the portfolio")

$$\beta_P = \frac{Cov(R_P, R_M)}{\sigma_M^2}$$

- Additional constraint: „**No short**"

$$w_{BL,i} \geq 0 \quad ,i = 1..N$$

(see, e.g., paper of K. Iordanidis); in Excel: requires additional calculations & solver constraints

Black-Litterman Approach - weights

Remark on treating weights w.r.t. absolute / relative Views

- The sum of portfolio weights has to add up to **100%**.
- **Purely *absolute* Views** are translated into independent *long* and *short* portfolios, thus causing portfolio weights to deviate from 100%. Therefore, normalization of weights is recommended.
- **Purely *relative* Views** are translated into related *long* and *short* portfolios, balancing each other so that portfolio weights still sum up to 100%.
- The use **of *absolute <u>and</u> relative* Views** again leads to portfolio weights deviating from 100% Therefore, again, normalization of weights is recommended.
- *Note that the "normalization" has to be included in the optimization process as a constraint.*

Black-Litterman Approach

Numerical example on treating weights w.r.t. absolute / relative Views

- The example is based on the View scenario given on slides 18 and 19.

- **Purely *absolute* View:** Weights add up to 110%, with all weights unchanged except for the asset (sector) CNYL under view (weight increases by 10%pts.).
  Normalization to 100% leads to weight changes in *all* positions!
  (Note: Lowering the expected return of CNYL to, e.g., 5,5% yields a total portfolio weight of only 86%. Again, weight normalization is recommended, spreading for the -14%pts across *all* weights.)

- **Purely *relative* Views:** Weights add up to 100%, with the weights of the viewed assets just offsetting the *long* and *short* positions.

- ***Absolute <u>and</u> relative* Views:** Weights add up to 129%, with *long* and *short* positions for the viewed assets as intuitively expected and the unviewed assets' weights remaining unchanged.
  Normalization to 100% consequently leads to weight changes in *all* positions!

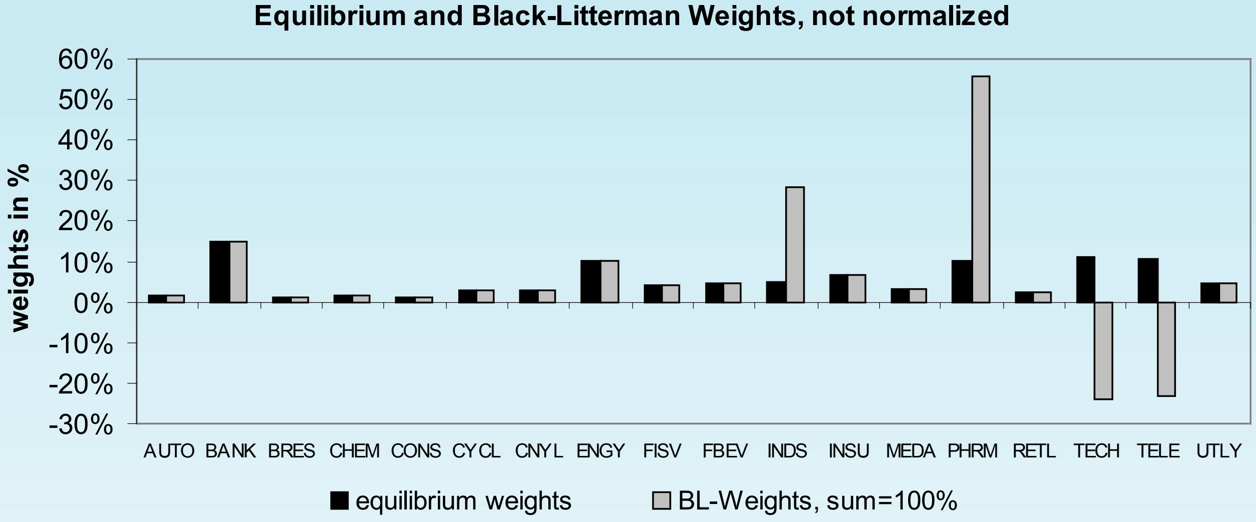
Example: DJ STOXX

Black-Litterman Approach

(cont.) non-normalized weights

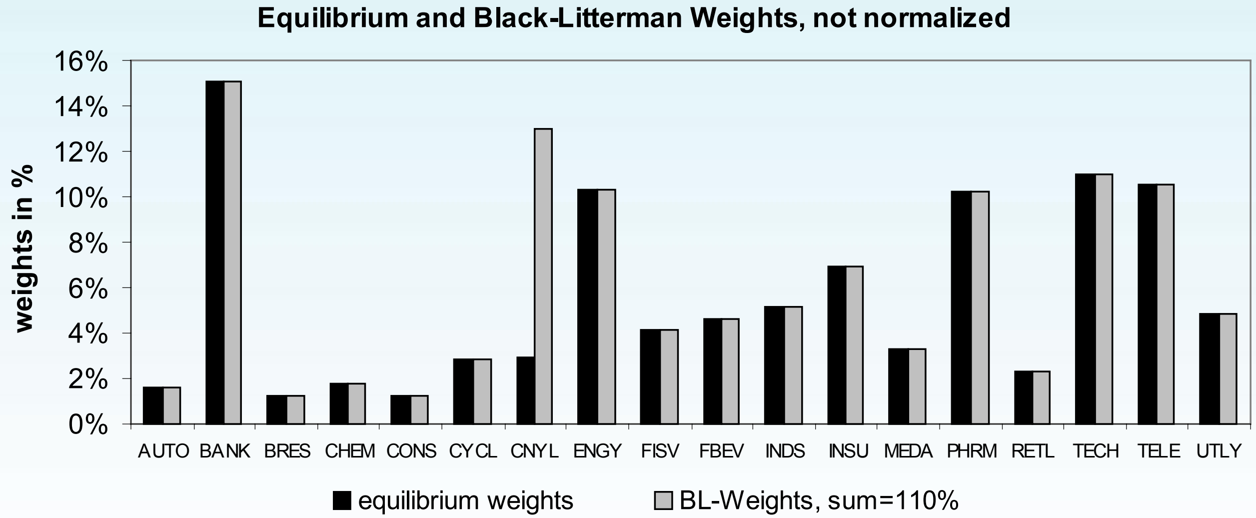
Purely relative Views

Σ weights = 100%,
no normalization required



Purely absolute Views

Σ weights = 110%,
normalization recommended



Black-Litterman Approach

Numerical example (cont.)

Relative <u>and</u> absolute Views

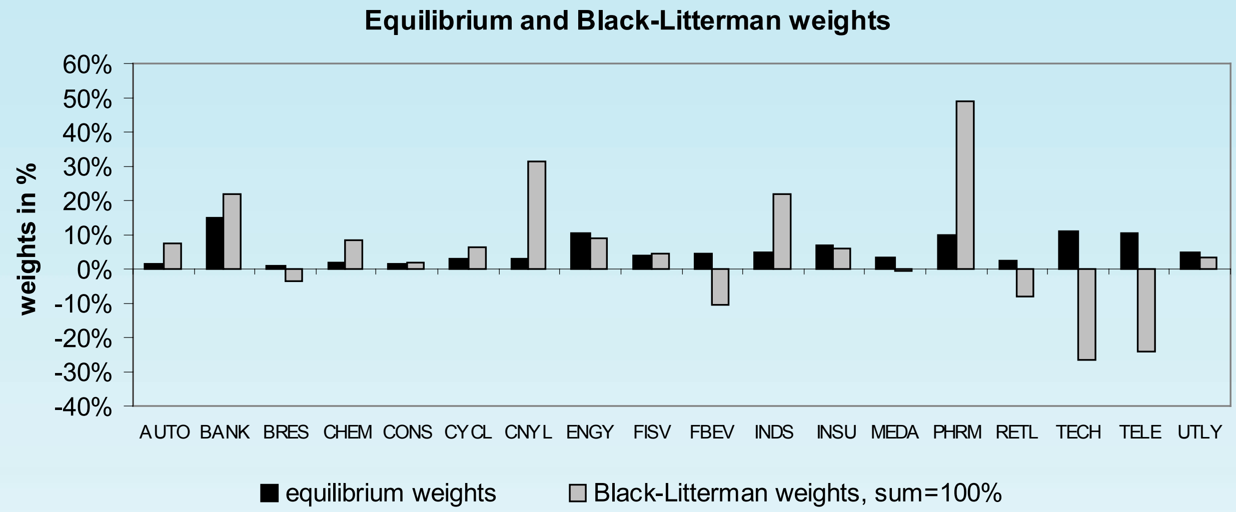
- weights normalized

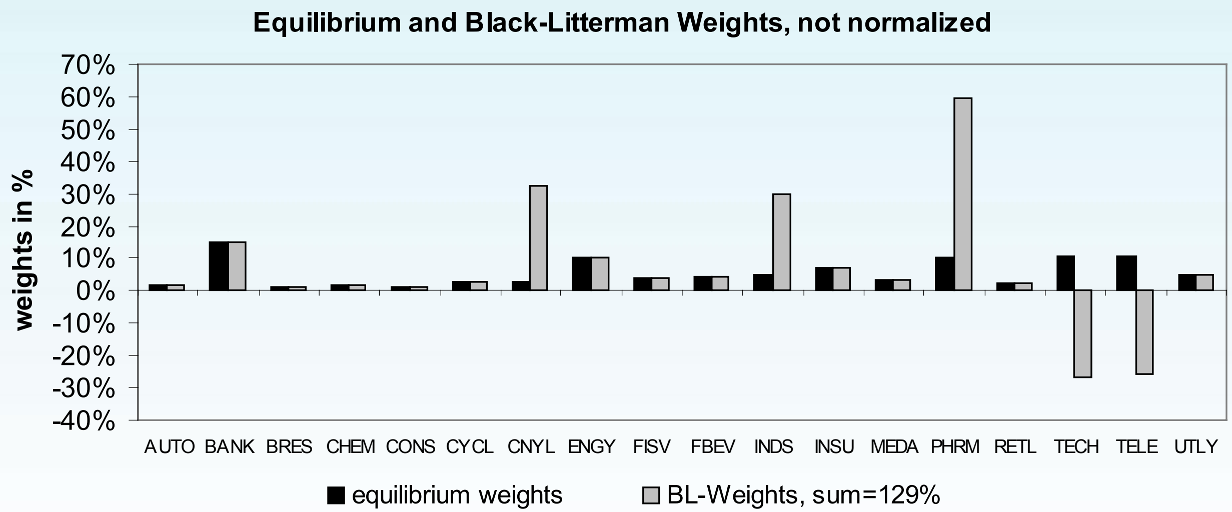
  Σ weights = 100%

and

- weights not normalized

  Σ weights = 129%



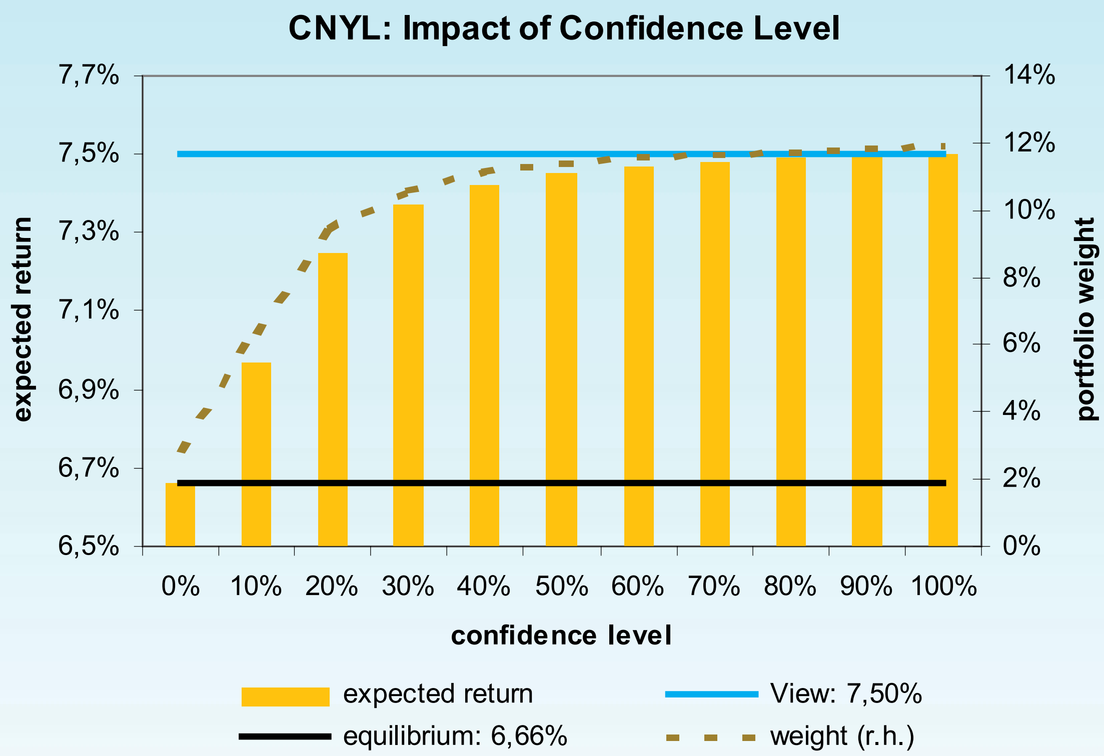


Black-Litterman Approach - confidence and CNYL

Influence of degree of confidence on BL-returns and BL-weights, I

Focus on asset „CNYL" only:

- Equilibrium return = 6.66%,
- Equilibrium weight = 2.90%
- View on return: 7.5% ± 1.5% (equivalent to a range of 6 - 9%)



Observations:

- Low confidence: → equilibrium return
- High confidence: Asymptotic approach to the View value of 7.5%.
- Limit: At a confidence level of 100%, BL fully accepts the strong view of 7.5%.
- Weights: from 2.9% (= market cap, due to confidence of 0% de facto *no view),* up to 12% (overweighting due to the *strong view* confidence of 100%).

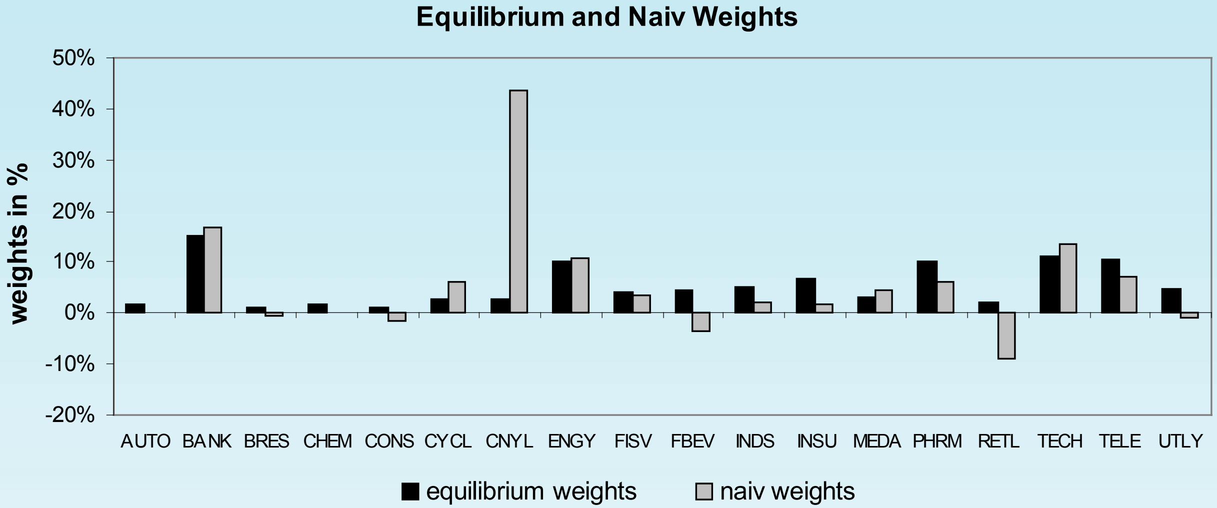
Black-Litterman Approach - weights of CNYL I

BL compared to straight MV

View on Return:

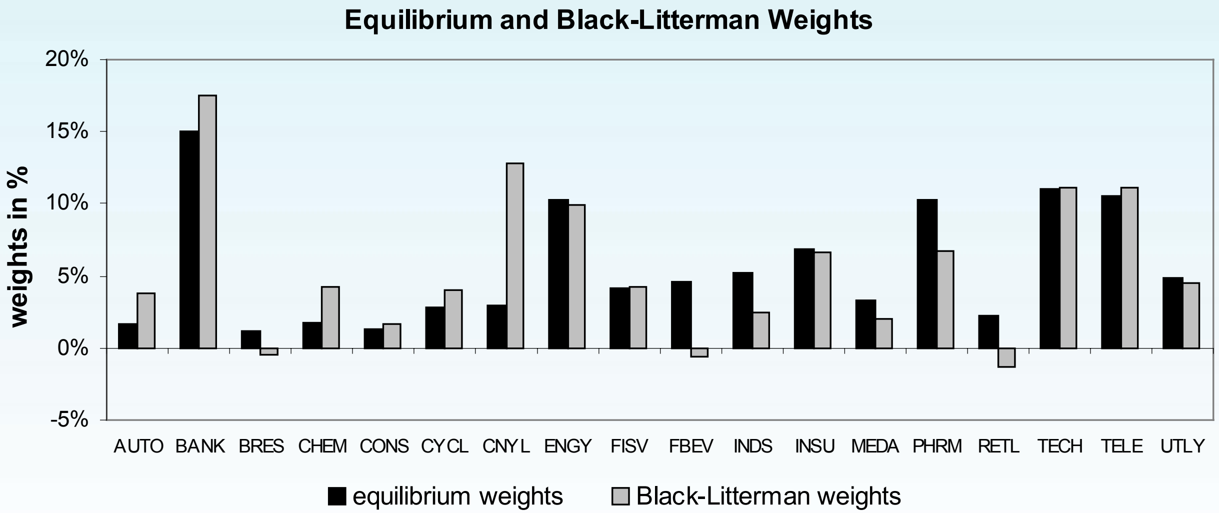
from 6.6% up to 7.5%

(with strong confidence)



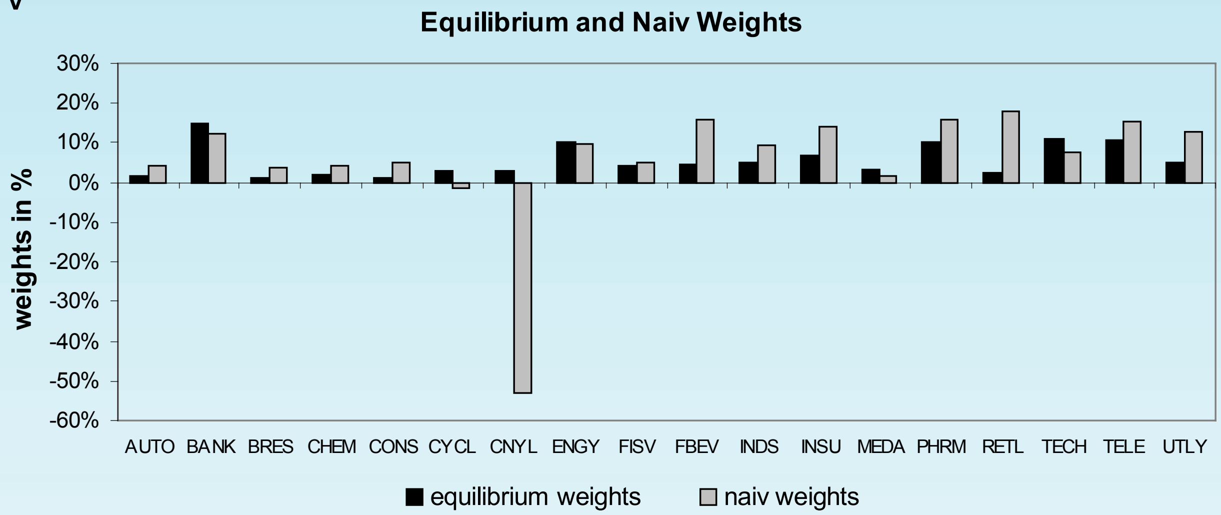
Result

- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach



Black-Litterman Approach - weights of CNYL II

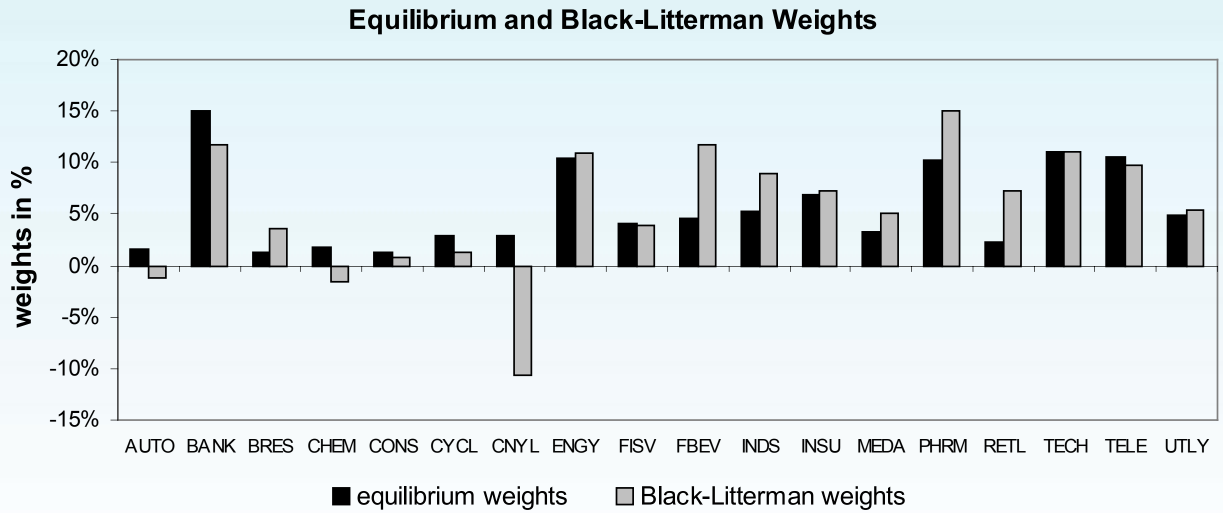
BL compared to straight MV



View on Return:

from 6.6% down to 5.5%

(with strong confidence)



Result

- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach

Black-Litterman Approach - confidence and weights

Influence of degree of confidence on BL-returns and BL-weights

Behavior of asset weights:

- complete portfolio, 18 sectors

Observations:

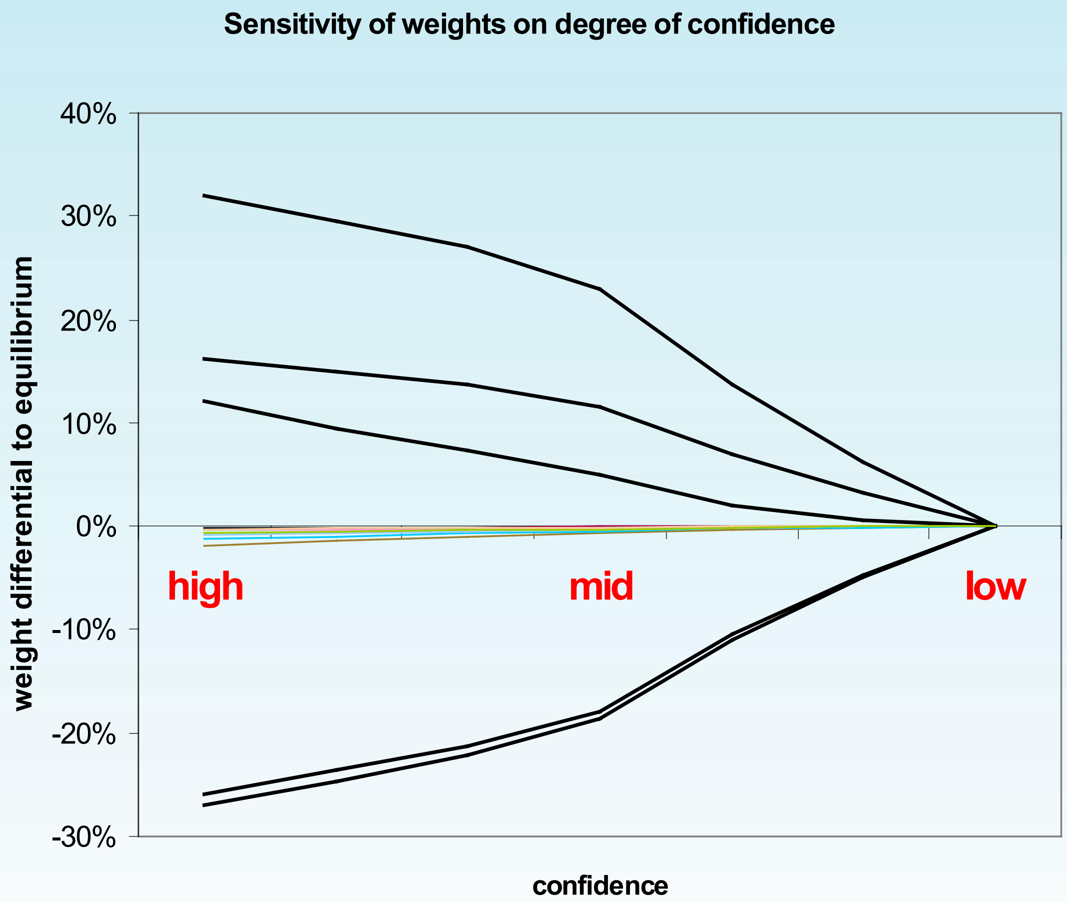
- Low degrees of confidence:

  BL-weights are close to weights in equilibrium (=market cap's).

- Higher degree of confidence:

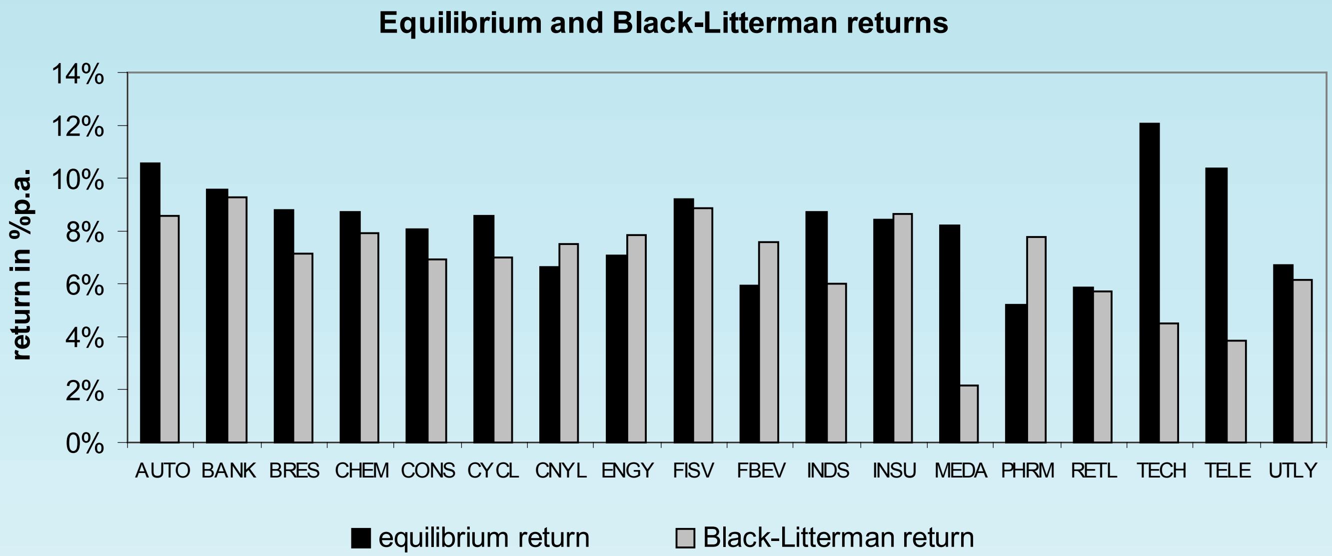
  Weights approach equilibrium values on either underweighting (*short*) or overweighting (*long*) path.

- Most significant weight changes for the assets under View.

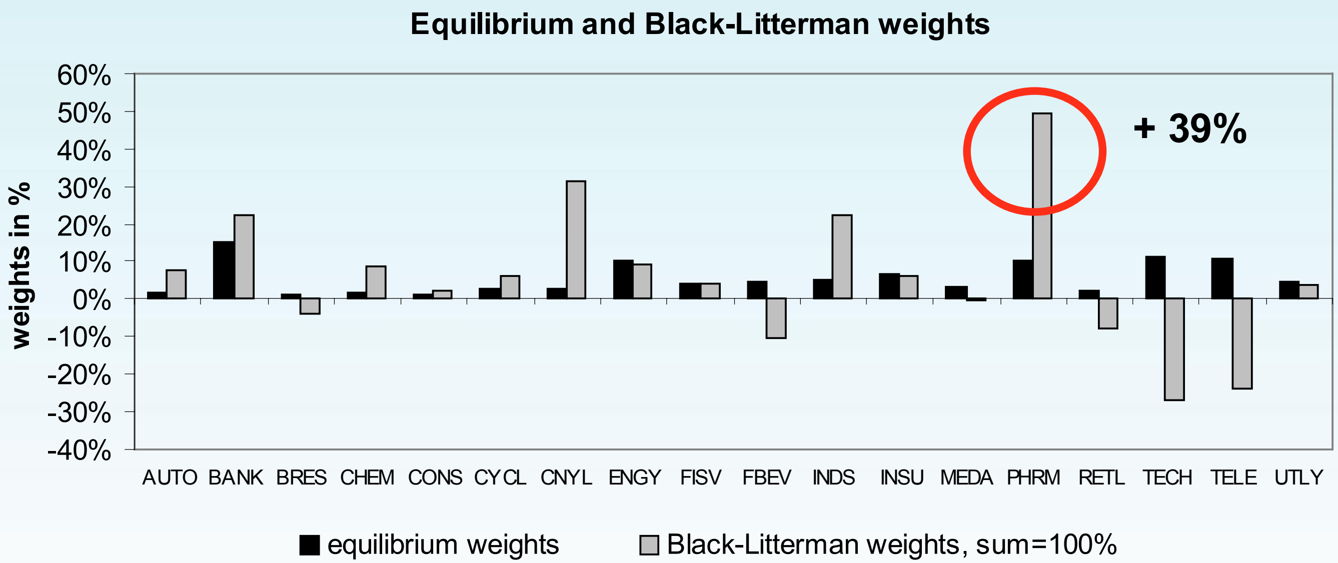


Black-Litterman Approach

Strong confidence

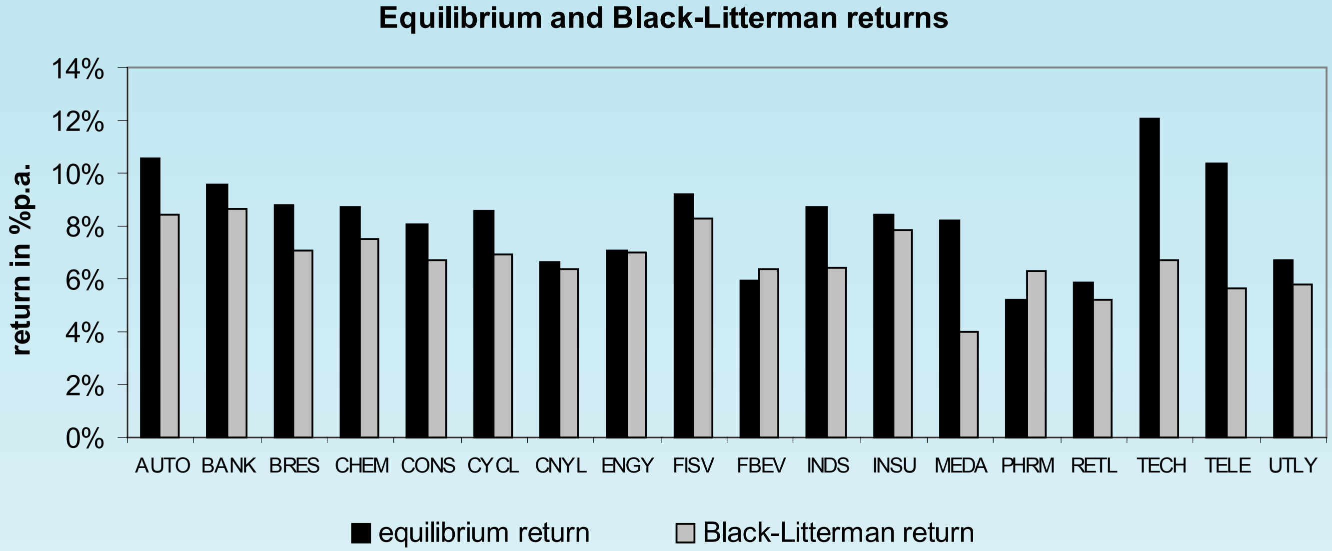


- Large changes in weights due to the "strong views"

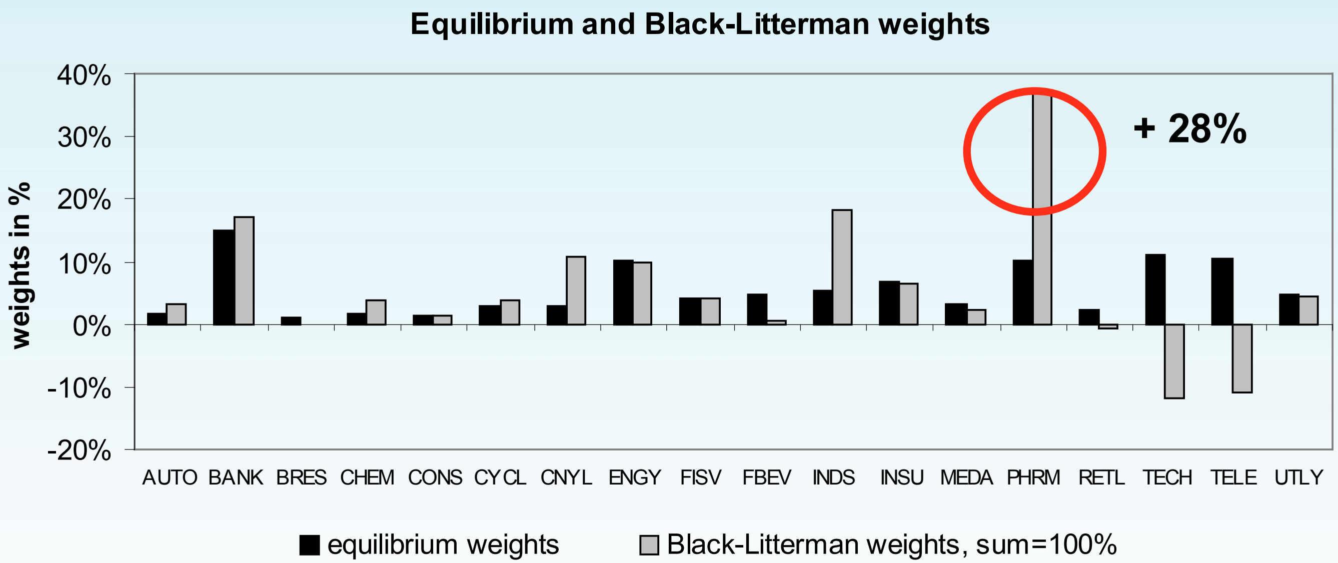


Black-Litterman Approach

Mid confidence

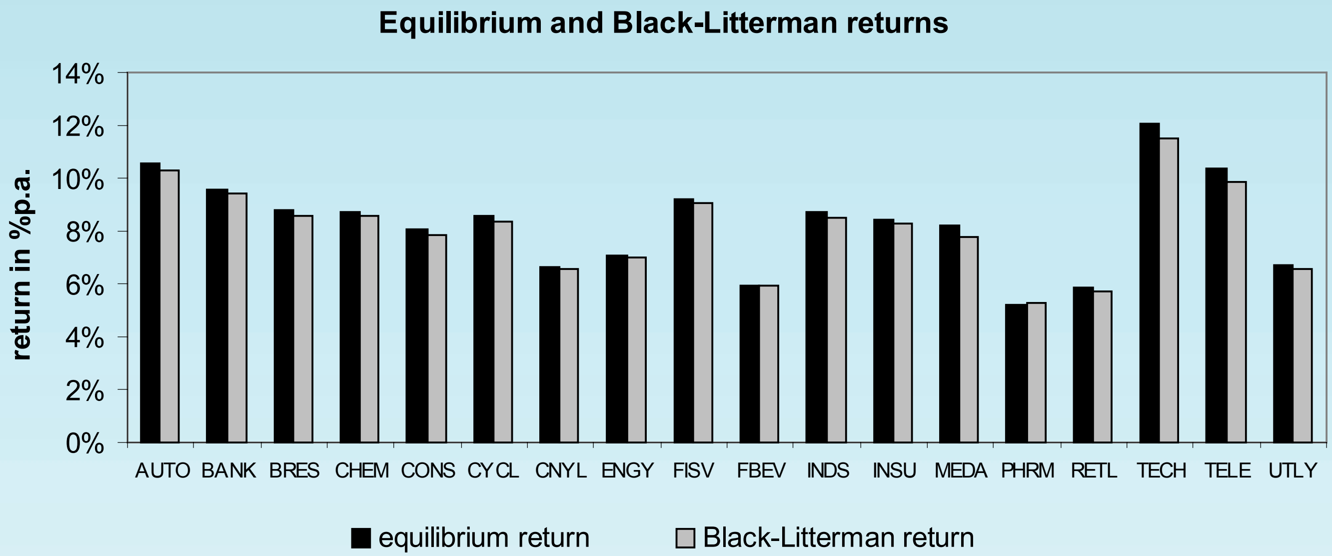


- Moderate changes in weights

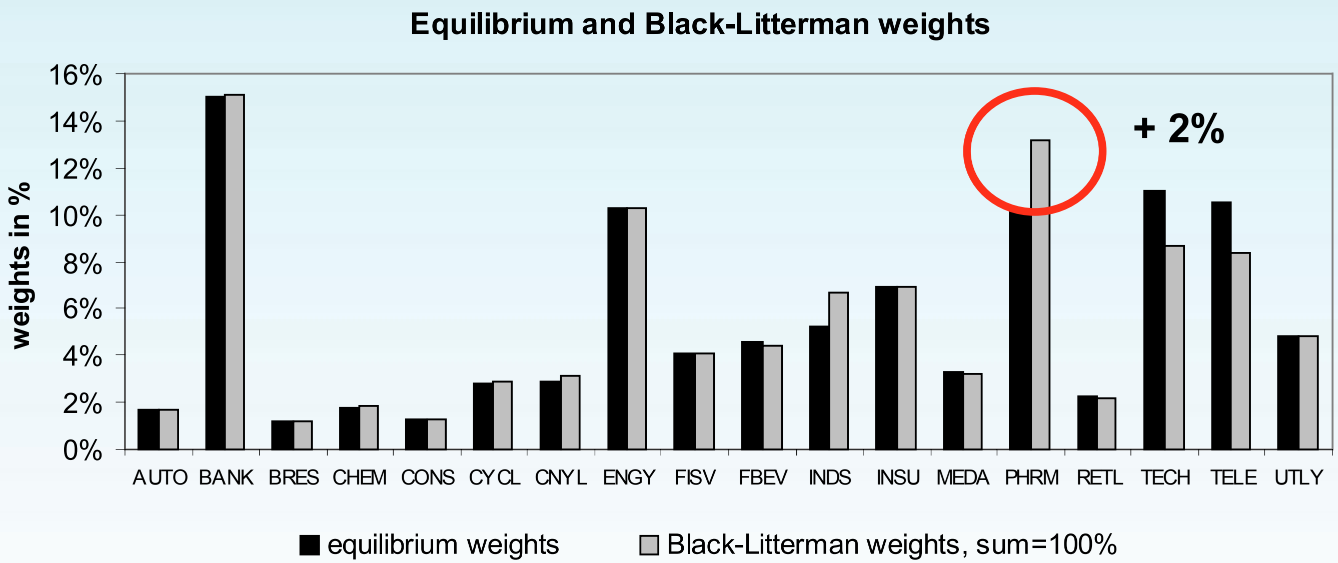


Black-Litterman Approach

Poor confidence



- Weights stay close to equilibrium weights



Black-Litterman Approach - Conclusion I

Traditional „Straight MV" vs „BL plus MV" approach

	straight MV	Black-Litterman → MV
▪ **Return estimates:**		
	o required for <u>each</u> asset	**required for <u>selected</u> assets**
	o assumed as <u>certain</u>	**degree of <u>confidence</u>**
	o <u>absolute</u> return figures	**<u>absolute or relative</u> Views**
	o c.p.	**consistent**
▪ **Reference return:**		
	o none	**equilibrium returns**

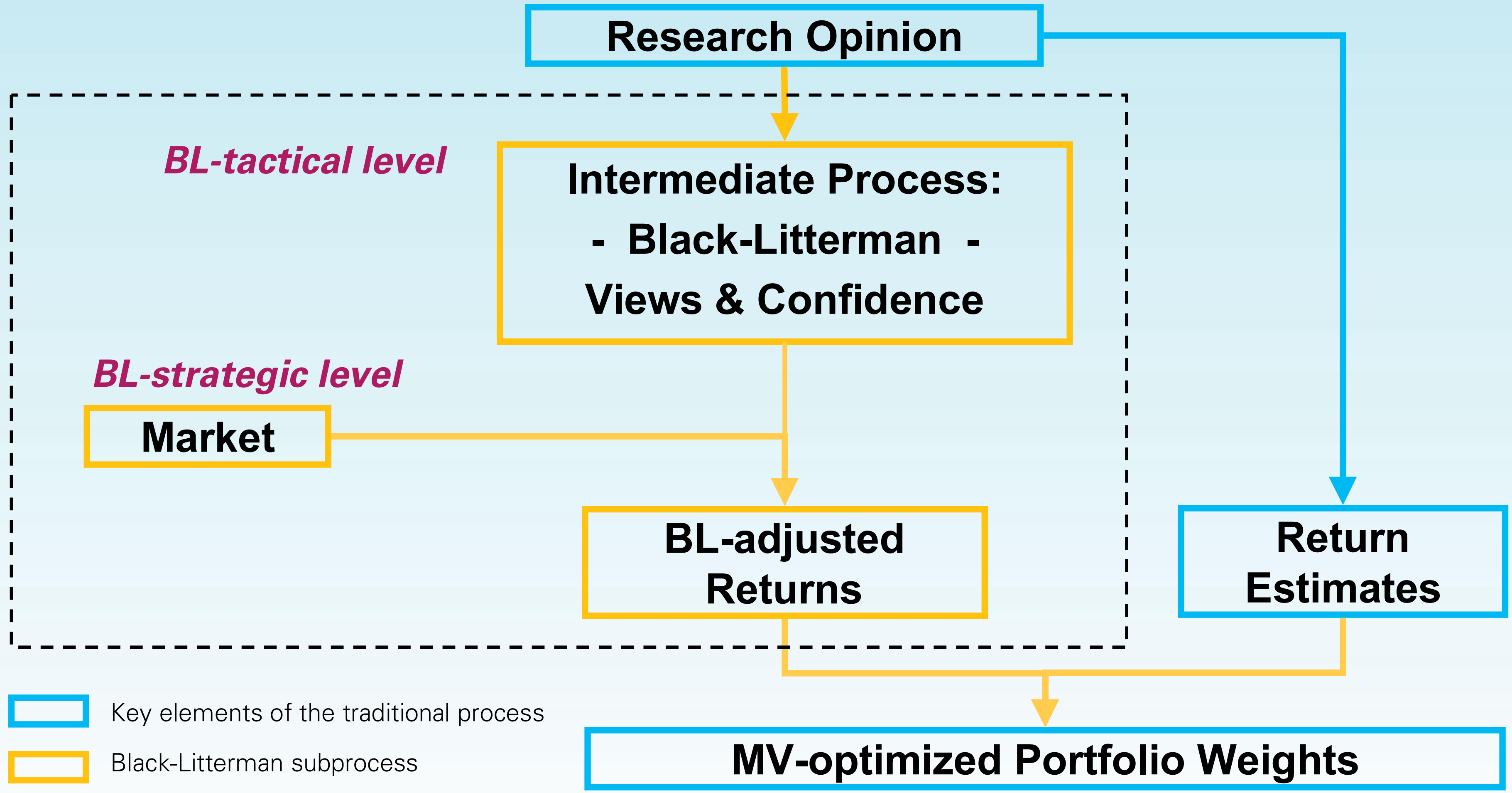
Black-Litterman Approach - Conclusion II

Traditional „Straight MV“ vs „BL plus MV“ approach

	straight MV	Black-Litterman → MV
■ **MV-optimized Portfolios:**		
	o extreme asset weights	**reliable asset weights**
	o changes in return estimates	
	⇒ huge weight fluctuations	⇒ **moderate weight changes**
	o portfolios unreliable	**consistent structure**
	o MV-results hardly accepted	**higher acceptance**
	o reflects c.p. opinions	**„correlated Views“**

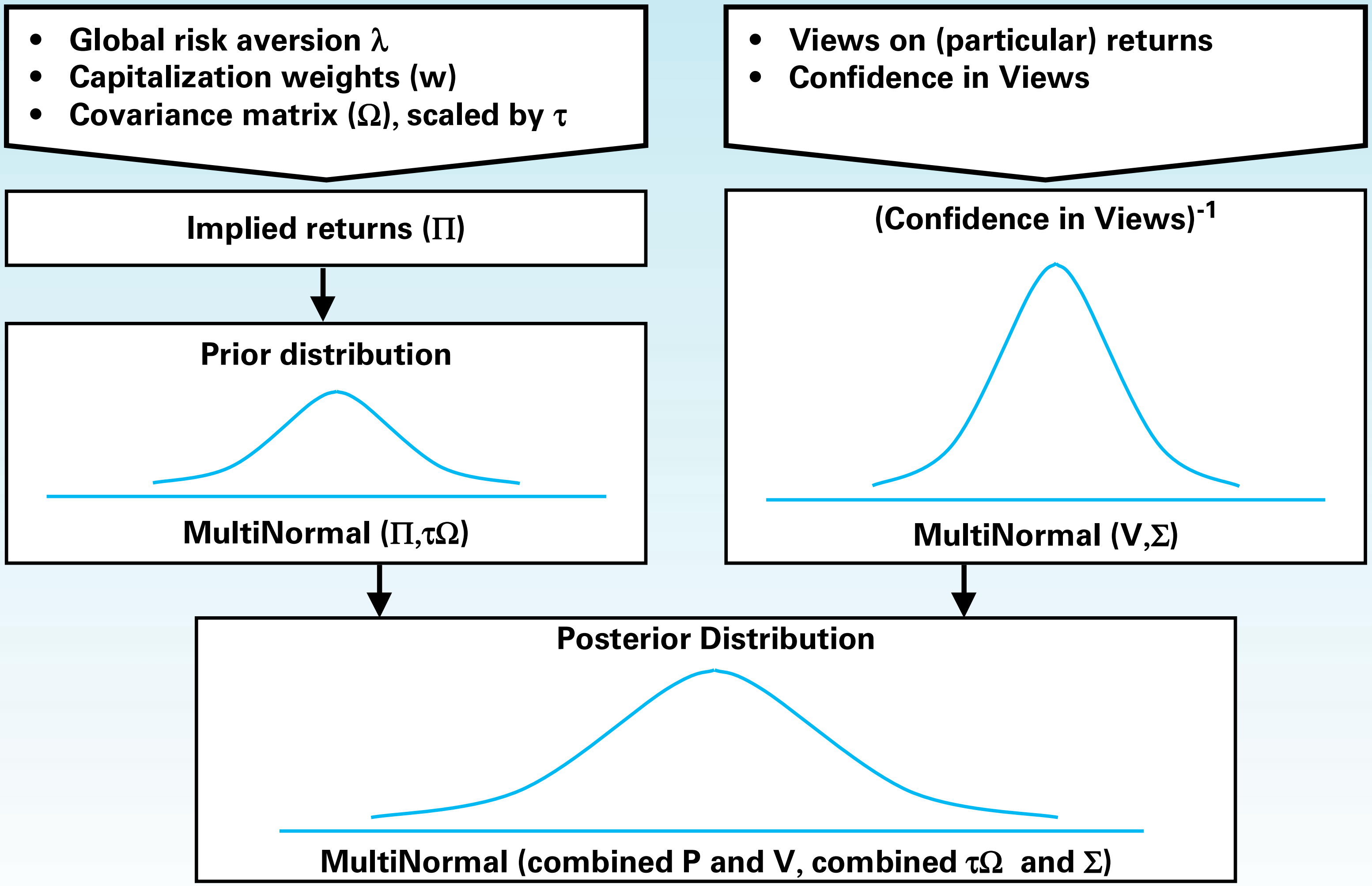
Black-Litterman Approach - Conclusion III

BL as a building-block of an enhanced asset allocation process



Black-Litterman Approach - Conclusion IV

... in a more formal way



Layout inspired by
K.Iordanidis

Black-Litterman Approach - more about it

Suggestions for further reading

- Black F. and Litterman R.: *Global Portfolio Optimization*, Fin.Analysts Journal, Sep.1992, p.28-43
- Black F. and Litterman R.: *Asset Allocation: Combining investor views with market equilibrium*, Goldman-Sachs, Fixed Income Research, Sep.1990
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