RISK-ADJUSTED PERFORMANCE ANALYSIS

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INTRODUCTION (1/2)

Common wisdom today: Performance is only a *biased* and *noisy* signal for the quality of asset management.

**BIAS:** Risk-return trade off

**NOISE:** Skill versus luck

Risk-adjusted performance analysis is about quantifying and analyzing unbiased performance. It can also be used to distinguish skill from luck.

*This presentation wants to summarize the best practice concepts and methods in risk-adjusted performance analysis. It is of a descriptive nature.*
INTRODUCTION (2/2)

CONSULTANTS SWITZERLAND

- Quantitative performance analysis as a criterion for manager selection has been practiced for about 5 years → a new toy
- Most often requested statistics: Sharpe and Information Ratio

STRUCTURED ALPHA™ (Watson Wyatt)

- **Alpha**: Net Fund Return – Net Benchmark Return. Net = Fees & switching costs
- **Sigma**: Tracking Error = Standard Deviation of Alpha
  → Financial Factors summarized in…
  *Investment Efficiency*: Net Alpha / TE = IR. Used to rank managers
- **Theta**: Non-financial factors are of importance to trustees: ‘Sleep Well’ factors (loss aversion), ‘Seems Good’ factors (brand names)
- There exists a trade off between financial and Theta factors. That’s why you need WW’s consulting service…
FRAMEWORK

1. CLIENT PREFERENCES

Client likes return, dislikes risk*: 

\[ U = U(\mu_p, \sigma_p) \]

\[ dU = \frac{\partial U}{\partial \mu_p} d\mu_p + \frac{\partial U}{\partial \sigma_p} d\sigma_p \]

\[ \frac{\partial U}{\partial \mu_p} > 0 \]

\[ \frac{\partial U}{\partial \sigma_p} < 0 \]

*risk is usually defined as the second moment of the return distribution.

2. BENCHMARKING

- **Client** chooses benchmark and sets targets/limits for alpha, beta a.s.o. at inception
- **Portfolio Mgt** controls alpha, beta and beta after inception

3. INDEX MODELS

\[ \mu_p - r_f = \alpha + \beta \cdot (\mu_B - r_f) + \varepsilon \]

Validity of index models to analyze performance largely depends on the implementation of benchmarking!
SHARPE RATIO (1/2) – DEFINITION

\[ S = \frac{\mu_p - r_f}{\sigma_p} \]

\( \mu_p \) ...Portfolio Return
\( r_f \) ...Riskfree Rate
\( \sigma_p \) ...Portfolio Volatility
SHARPE RATIO (2/2) - APPLICATION

MEASUREMENT

- Annualized portfolio return, portfolio volatility
- Annualized risk-free rate
  - Choice is important because it can change ranking
  - Problematic in an international context
- Aggregation
  - No straight-forward adding-up because of covariance effects between volatilities
- Are negative values ambiguous?

\[
\begin{align*}
\frac{\mu_p - r_f}{\sigma_p} & \quad + \quad \frac{\mu_p - r_f}{\sigma_p} \\
\frac{\mu_p - r_f}{\sigma_p} & \quad - \quad \frac{\mu_p - r_f}{\sigma_p}
\end{align*}
\]

INTERPRETATION

- Summary of the first two moments of the portfolio excess return distribution. Model-free
- Suitable for comparisons across asset classes
- Target in Mean-Variance Optimization
- Does not assume a benchmark. Implicit benchmark is risk-free rate.
- Statistical hypothesis testing: test for non-zero performance
  \[ t-\text{Stat} = S \times \sqrt{T} \]
TREYNOR RATIO (1/2) - DEFINITION

\[ T = \frac{\mu_P - r_f}{\beta_P} \]

\( \mu_P \) ...Portfolio Return

\( r_f \) ...Riskfree Rate

\( \beta_P \) ...Portfolio Beta

\[ \beta = \frac{\sigma_{PB}^2}{\sigma_B^2} = \rho_{PB} \frac{\sigma_P}{\sigma_B} \]

\( \sigma_{PB} \) ...Covariance

\( \rho_{PB} \) ...Correlation
## TREYNOR RATIO (2/2) - APPLICATION

<table>
<thead>
<tr>
<th><strong>MEASUREMENT</strong></th>
<th><strong>INTERPRETATION</strong></th>
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<tbody>
<tr>
<td>▪ Annualized portfolio return, annualized risk-free rate</td>
<td>▪ Accounts for systematic and unsystematic risk (CAPM-based): Only systematic risk is considered.</td>
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<tr>
<td>▪ Estimation of beta can be distorted by market timing. Extensions: Squared regression, H/M regression</td>
<td>▪ Comparison across different asset classes problematic (beta is dependent on benchmark)</td>
</tr>
<tr>
<td>▪ Aggregation: Straight-forward. Beta of aggregate is weighted sum of constituent’s betas</td>
<td>▪ Choice of benchmark affects ranking</td>
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INFORMATION RATIO (1/3) - DEFINITION

\[ IR = \frac{\alpha_P}{TE_P} \]

\( \alpha_P \)  \text{ ...Portfolio Alpha}

\( TE_P \)  \text{ ...Portfolio Tracking Error}

**Active Portfolio Return: Alpha**
- Average annual performance
- Jensen’s Alpha

Choice should be consistent to choice of TE definition…
IR (2/3) - TRACKING ERROR DEFINITIONS

\[ \mu_P = \alpha + \beta \cdot \mu_B + \varepsilon \]

\[ \sigma_P^2 = \beta^2 \cdot \sigma_B^2 + \sigma_\varepsilon^2 = \rho_{PB}^2 \cdot \frac{\sigma_P^2}{\sigma_B^2} \cdot \sigma_B^2 + \sigma_\varepsilon^2 \quad \text{with} \ldots \quad \beta = \frac{\sigma_{PB}}{\sigma_B^2} = \rho_{PB} \frac{\sigma_P}{\sigma_B} \]

\[ \sigma_P^2 = \rho_{PB}^2 \cdot \sigma_P^2 + \sigma_\varepsilon^2 \quad \text{TE}_P = \sigma_P \cdot \sqrt{1 - \rho_{PB}^2} = \sigma_\varepsilon \]

...Residual risk = Risk uncorrelated with BM.

\[ \text{TE}_P = \sqrt{Var(r_P - r_B)} \quad \text{...Standard deviation of performance} \]

\[ \mu_P - \mu_B = \alpha + \beta \cdot \mu_B + \varepsilon - \mu_B = \alpha + (\beta - 1) \cdot \mu_B + \varepsilon \]

\[ \sqrt{Var(\mu_P - \mu_B)} = \sqrt{(\beta - 1)^2 \cdot \sigma_B^2 + \sigma_\varepsilon^2} \quad \text{→ For \( \beta \neq 1 \), the Stdev(perf) is always larger than residual risk} \]

\[ \text{→ Stdev(perf) depends on benchmark volatility} \]
IR (3/3) - APPLICATION

MEASUREMENT

- Measurement of Alpha & TE with index or factor models makes IR dependent on model specification errors.

INTERPRETATION

- Summary statistic: Active return / active risk trade off, efficiency ratio
- Fundamental Law of Active Mgt:
  \[ IR_{\text{ex ante}} = IC \times BR \]
  
  IC: Information Coefficient 
  \[ Corr(\text{Forecast } r, \text{Actual } r) \]
  
  BR: Breadth of strategy 
  \# of independent bets taken
- Statistical hypothesis testing: Non-zero alpha signals
  \[ t-\text{Stat} = IR \times \sqrt{T} \]
- Generally not consistent with MVO…
M MEASURES - M² (1/2)

\[ \mu_{RAP} = \frac{\sigma_B}{\sigma_P} (\mu_P - r_f) + r_f \]

\( \mu_{RAP} \) ...Risk - Adjusted Return
\( \sigma_P \) ...Portfolio Volatility
\( \sigma_B \) ...Benchmark Volatility
\( \mu_f \) ...Portfolio Return
\( r_f \) ...Riskfree Rate

→ Performance is volatility-adjusted by leveraging the fund with risk-free-investments so that the resulting volatility equals the benchmark volatility.

\[ \frac{\sigma_B}{\sigma_P} \] ...Leverage Factor \( d \)

\[ \mu_{RAP} = d \cdot \mu_P + (1 - d) \cdot r_f \]
M MEASURES - M² (2/2)

- The difference between M² can be interpreted intuitively: Unit of measurement is % → Risk expressed in units of return
- M² rankings are independent of the chosen benchmark (benchmark risk as a scaling factor)
- The M² measure is a transformed Sharpe Ratio and therefore consistent with MPT

\[ \mu_{RAP} = \frac{\sigma_B}{\sigma_P} (\mu_p - r_f) + r_f = \sigma_B \cdot S + r_f \]

- M² ranking equals Sharpe Ratio ranking
- Drawback: Correlation risk (timing, selection) is neglected…
M MEASURES - M³ (1/2)

\[ \mu_{CAP} = a \cdot \mu_p + b \cdot \mu_B + (1-a-b) \cdot r_f \]

\[ \bar{\rho}_{PB} = 1 - \frac{\text{TE}_{PB}^2}{2 \cdot \sigma_B^2} \]

\[ a = \frac{\sigma_B}{\sigma_p} \sqrt{\frac{(1-\bar{\rho}_{PB})^2}{(1-\rho_{PB})^2}} \]

\[ b = \bar{\rho}_{PB} - \rho_{PB} \sqrt{\frac{(1-\bar{\rho}_{PB})^2}{(1-\rho_{PB})^2}} \]

→ M³ cannot be illustrated graphically in an elegant way (three dimensions)

→ Performance is correlation-adjusted by leveraging the fund with active, passive and risk-free funds so that (1) the resulting volatility equals benchmark volatility and (2) the TE equals the Target TE
M MEASURES - $M^3$ (2/2)

- $M^3$ is ‘volatility-risk- and-correlation-risk’-adjusted-performance
- $M^3$ rankings differ from $M^2$ and rankings
- If no target tracking error exists, $a = 0$ and $M^3$ will equal $M^2$
- $M^3$ can be used in a forward looking sense: It can provide ex ante guidance how to structure portfolios with TE restrictions (given the stability of distributional characteristics in the future)
- Drawback (of all RAP measures): Timing and selection activities are not decoupled.
RAPP (1/2) …Risk-Adjusted Performance and Positioning Index

\[ U(\mu_p, \sigma_p) \approx U(\mu_B, \sigma_B) + \frac{\partial U}{\partial \mu} d\mu + \frac{\partial U}{\partial \sigma} d\sigma \]

\[ RAPP \equiv \frac{U(\mu_p, \sigma_p) - U(\mu_B, \sigma_B)}{\partial U / \partial \mu} \approx \alpha + \lambda \cdot TE \]

\[ \lambda = \frac{\partial U / \partial \sigma}{\partial U / \partial \mu} \quad \text{...Risk Aversion} \]

\[ \alpha \approx d\mu \]

\[ TE \approx d\sigma \]
RAPP (2/2)

- The RAPP concept is very flexible (TE targets, for example)

- Utility functions are considered at least problematic by many economists, especially in decision making under risk (‘Homo Oeconomicus’ debate, Behavioral Finance)

- To implement RAPP, the marginal utilities of parameters (risk aversion, for example) have to be quantified. RAPP ranking will depend on these marginal utilities.

- Aggregation across asset classes is achieved by measuring everything in terms of utilities. A new aggregation problem is introduced: aggregating client preferences.

- Non-financial aspects are neglected. Considering the importance of such factors: Is it worth developing and maintaining an internal RAP measure?
RELATIONSHIP BETWEEN MEASURES

Markets: S&P 500, DJ Euro STOXX 50, SPI, MSCI Japan, FTSE 100

Observations:
- RAP strategies are highly correlated
- The ex ante / ex post choice of RAP targets creates significant incentives
DISCUSSION

IT’S YOUR TURN...